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Chapter 5: Simultaneous Linear Equations (3 weeks)

Utah Core Standard(s):
- Analyze and solve pairs of simultaneous linear equations. (8.EE.8)
  a) Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
  b) Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.
  c) Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

Academic Vocabulary: system of linear equations in two variables, simultaneous linear equations, solution, intersection, ordered pair, elimination, substitution, parallel, no solution, infinite solutions

Chapter Overview:
In this chapter we discuss intuitive, graphical, and algebraic methods of solving simultaneous linear equations; that is, finding all pairs (if any) of numbers $(x, y)$ that are solutions of both equations. We will then use these understandings and skills to solve real world problems leading to two linear equations in two variables.

Connections to Content:
Prior Knowledge: In chapter 1, students learned to solve one-variable equations using the laws of algebra to write expressions in equivalent forms and the properties of equality to solve for an unknown. They solved equations with one, no, and infinite solutions and studied the structure of an equation that resulted in each of these outcomes. In chapter 3, students learned to graph and write linear equations in two-variables. Throughout, students have been creating equations to model relationships between numbers and quantities.

Future Knowledge: In subsequent coursework, students will gain a conceptual understanding of the process of elimination, examining what is happening graphically when we manipulate the equations of a linear system. They will also solve systems that include additional types of functions.
Kevin and Nina are competing in a bike race. When Kevin is ninety miles into the race, he is in first place. Nina is in second place and is 15 miles behind Kevin.

a. From this point, Kevin continues the race at a constant rate of 25 mph and Nina continues the race at a constant rate of 30 mph. When will Nina catch Kevin? Solve this problem using any method you wish.

b. If the race is 150 miles long, who will win? Assume Nina and Kevin bike at the speeds given in part a).

c. Now suppose the following: Ninety miles into the race, Kevin is still in first place and Nina is still in second place, 15 miles behind Kevin. But now Kevin and Nina both finish out the race at a speed of 30 mph. When will Nina catch Kevin? If the race is 150 miles long, who will win?

The goal of this problem is that students will have the opportunity to explore a problem that can be solved using simultaneous linear equations from an intuitive standpoint, providing insight into graphical and algebraic methods that will be explored in the chapter. Students also gain insight into the meaning of the solution(s) to a system of linear equations. This problem requires students to analyze givens, constraints, relationships, and goals. Students may approach this problems using several different methods: picture, bar model, guess and check, table, equation, graph, etc.

The student officers are buying packs of streamers and balloons to decorate for a school dance. Packs of balloons cost $3.50 and packs of streamers cost $2. If the student officers have a total of 12 packs of decorations and spent $31.50, how many packs of balloons did they buy? How many packs of streamers did they buy? Write the solution in a complete sentence.

The ability to create and solve equations gives students the power to solve many real world problems. They will apply the strategies learned in this chapter to solve problems arising in everyday life that can be modeled and solved using simultaneous linear equations.
| Look for and make use of structure | One equation in a system of linear equations is \(6x + 4y = -12\).  
  
  a. Write a second equation for the system so that the system has only \textit{one solution}.  
  
  b. Write a second equation for the system so that the system has \textit{no solution}.  
  
  c. Write a second equation for the system so that the system has \textit{infinite solutions}.  
  
  \textit{In this problem, students may analyze the structure of the first equation in order to discern possible second equations that will result in one, infinitely many, or no solution.} |
|---|---|
| Use appropriate tools strategically. | A farmer saw some chickens and pigs in a field. He counted 60 heads and 176 legs. Determine exactly how many chickens and pigs he saw.  
  
  a. Solve the problem using the methods strategies studied in this chapter.  
  
  b. Which method do you prefer using to solve this problem? Use your preferred method to determine the number of chickens and pigs in a field with 45 heads and 146 legs.  
  
  \textit{While solving this problem, students should be familiar with and consider all possible tools available: graphing calculator, graph paper, concrete models, tables, equations, etc. Students may gravitate toward the use of a graphing calculator given the size of the numbers. This technological tool may help them to explore this problem in greater depth.} |
5.0 Anchor Problem: Chickens and Pigs

A farmer saw some chickens and pigs in a field. He counted 30 heads and 84 legs. Determine exactly how many chickens and pigs he saw. Solve the problem in as many different ways as you can and show your strategies below.
Section 5.1: Understand Solutions of Simultaneous Linear Equations

Section Overview:
In this section, students are solving simultaneous linear equations that have one, no, or infinite solutions using intuitive and graphical methods. In order to access the problems initially students may use logic, and create pictures, bar models, and tables. They will solve simultaneous linear equations using a graphical approach, understanding that the solution is the point of intersection of the two graphs. Students will understand what it means to solve two linear equations, that is, finding all pairs (if any) of numbers \((x, y)\) that are solutions of both equations and they will interpret the solution in a context.

Concepts and Skills to Master:
*By the end of this section, students should be able to:*

1. Solve simultaneous linear equations by graphing.
2. Understand what it means to solve a system of equations.
3. Identify and provide examples of systems of equations that have one solution, infinitely many solutions, or no solution.
4. Interpret the solution to a system in a context.
5.1a Class Activity: The Bake Sale
1. The student council is planning a bake sale to raise money for a local food pantry. They are going to be making apple and peach pies. They have decided to make 10 pies. Each pie requires 2 pounds of fruit; therefore they need a total of 20 pounds of fruit.
   a. In the table below, fill out the first two columns only with 8 possible combinations that will yield 20 pounds of fruit.

<table>
<thead>
<tr>
<th># of Pounds of Apples</th>
<th># of Pounds of Peaches</th>
<th>Cost of Apples</th>
<th>Cost of Peaches</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
</tbody>
</table>

b. One pound of apples costs $2 and one pound of peaches cost $1. Fill out the rest of the table above to determine how much the student council will spend for each of the combinations.

c. Mrs. Harper, the student council advisor, tells the students they have exactly $28 to spend on fruit. How many pounds of each type of fruit should they buy so that they have the required 20 pounds of fruit and spend exactly $28?
d. If \( a \) represents the number of pounds of apples purchased and \( p \) represents the number of pounds of peaches purchased, the situation above can be modeled by the following equations:

\[
\begin{align*}
a + p & = 20 \\
2a + p & = 28
\end{align*}
\]

Write in words what each of these equations represents in the context.

\[a + p = 20\]  ______________________________

\[2a + p = 28\]  ______________________________

e. Does the solution you found in part c) make both equations true?

f. Graph the equations from part d) on the coordinate plane below. Label the lines according to what they represent in the context.

![Graph](image)

g. Find the point of intersection in the graph above. What do you notice?
h. The Bake Sale problem can be modeled and solved using a **system of linear equations**. Write in your own words what a **system of linear equations** is.

i. Explain, in your own words, what the **solution** to a system of linear equations is. How can you find the solution in the different representations (table, graph, equation)?

j. Josh really likes apple pie so he wants to donate enough money so that there are an equal number of pounds of peaches and apples. How much does he need to donate?
**5.1b Class Activity: Who Will Win the Race?**

1. Kevin and Nina are competing in a bike race. When Kevin is ninety miles into the race, he is in first place. Nina is in second place and is 15 miles behind Kevin.
   a. From this point, Kevin continues the race at a constant rate of 25 mph and Nina continues the race at a constant rate of 30 mph. When will Nina catch Kevin? Solve this problem using any method you wish.
b. If the race is 150 miles long, who will win? Assume Nina and Kevin bike at the speeds given in part a).

c. Now suppose the following: Ninety miles into the race, Kevin is still in first place and Nina is still in second place, 15 miles behind Kevin. But now Kevin and Nina both finish out the race at a constant speed of 30 mph. When will Nina catch Kevin? If the race is 150 miles long, who will win?
2. The graph below shows the amount of money Alexia and Brent have in savings.

   a. Write an equation to represent the amount \( y \) that each person has in savings after \( x \) weeks:

      Alexia: _____________________  Brent: _____________________

   b. Tell the story of the graph. Be sure to include what the point of intersection means in the context.
5.1b Homework: Who Will Win the Race

1. Gabriela and Camila like to race each other. Gabriela can run 10 feet/second while Camila can run 12 feet/second. Being a good sport, Camila gives Gabriela a 20-foot head start.
   a. How long will it take Camila catch Gabriela?

   b. If the girls are racing to a tree that is 30 yards away, who will win the race? (Remember there are 3 feet in 1 yard).

2. Darnell and Lance are both saving money. Darnell currently has $40 and is saving $5 each week. Lance has $25 and is saving $8 each week.
   a. When will Darnell and Lance have the same amount of money?

   b. How much will each boy have when they have the same amount of money?

   c. If both boys continue saving at this rate, who will have $100 first?
3. The graph below shows the amount of money Charlie and Dom have in savings.

   a. Write an equation to represent the amount \( y \) that each person has in savings after \( x \) weeks:
      
      Charlie: _____________________  
      Dom: _______________________  

   b. Tell the story of the graph.
5.1c Classwork: Solving Simultaneous Linear Equations by Graphing

1. **Skill Review:** Put the following equations into slope-intercept form.
   
   a. \(2x + y = 4\)  
   
   b. \(4x + 2y = -12\)  
   
   c. \(4y - x = 16\)  
   
   d. \(4x - 2y = -24\)  
   
   e. \(-y = x - 2\)  
   
   f. \(-2x + 5y = 3\)

2. Consider the linear equations \(2x + y = 4\) and \(y = 4x - 2\). Graph both equations on the coordinate plane below.

   a. Find the coordinates \((x, y)\) of the point of intersection.

   b. Verify that the point of intersection you found satisfies both equations.

---

The **solution(s) to a pair of simultaneous linear equations** is all pairs (if any) of numbers \((x, y)\) that are solutions of both equations, that is \((x, y)\) satisfy both equations. When solved graphically, the solution is the point or points of intersection (if there is one).
3. Determine whether \((3, 8)\) is a solution to the following system of linear equations:
\[
\begin{align*}
2x + y &= 14 \\
x + y &= 11
\end{align*}
\]

4. Determine whether \((0, -5)\) is a solution to the following system of linear equations:
\[
\begin{align*}
y &= 2x - 5 \\
4x + 5y &= 25
\end{align*}
\]

5. Consider the equations \(y = -2x\) and \(y = -\frac{1}{2}x - 3\). Graph both equations on the coordinate plane below and find the solution. Verify that the solution satisfies both equations.

6. Consider the equations \(-2x + y = -1\) and \(y = 2x + 4\). Graph both equations on the coordinate plane below and solve the system of linear equations.
7. Consider the equations $x + y = 3$ and $3x + 3y = 9$. Graph both equations on the coordinate plane below and solve the system of linear equations.

![Graph of linear equations](image)

8. In your own words, explain what the **solution to a system of linear equations** is.

9. In the table below, draw an example of a graph that represents the different solving outcomes of a system of linear equations:

<table>
<thead>
<tr>
<th>One Solution</th>
<th>No Solution</th>
<th>Infinitely Many Solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Graph" /></td>
<td><img src="image" alt="Graph" /></td>
<td><img src="image" alt="Graph" /></td>
</tr>
</tbody>
</table>
10. Without graphing, determine whether the following systems of linear equations will have one solution, no solution, or infinitely many solutions.

a. \( y = 8x + 2 \) and \( y = -4x \)  
   b. \( y = -\frac{2}{3}x - 5 \) and \( y + \frac{2}{3}x = 1 \)

c. \( 2x + y = 8 \) and \( y = 2x - 2 \)  
   d. \( x + y = 5 \) and \( -2x - 2y = -10 \)

e. \( 3x + 2y = 5 \) and \( 3x + 2y = 6 \)  
   f. \( y = 2x + 5 \) and \( 4x - 2y = -10 \)

11. One equation in a system of linear equations is \( 6x + 4y = -12 \).
   a. Write a second equation for the system so that the system has only one solution.

b. Write a second equation for the system so that the system has no solution.

c. Write a second equation for the system so that the system has infinite solutions.
1. Solve the system of linear equations graphically. If there is one solution, verify that your solution satisfies both equations.

<table>
<thead>
<tr>
<th></th>
<th>a. ( y = 3x + 1 ) and ( x + y = 5 )</th>
<th>b. ( y = -5 ) and ( 2x + y = -3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><img src="image1.png" alt="Graph a" /></td>
<td><img src="image2.png" alt="Graph b" /></td>
</tr>
<tr>
<td></td>
<td>c. ( y = -3x + 4 ) and ( y = \frac{1}{2}x - 3 )</td>
<td>d. ( x - y = -2 ) and ( -x + y = 2 )</td>
</tr>
<tr>
<td></td>
<td><img src="image3.png" alt="Graph c" /></td>
<td><img src="image4.png" alt="Graph d" /></td>
</tr>
</tbody>
</table>
e. \( y = \frac{1}{2}x - 2 \) and \( y = \frac{1}{2}x + 4 \)

f. \( 2x - 8y = 6 \) and \( x - 4y = 3 \)

g. \( y = 6x - 6 \) and \( y = 3x - 6 \)

h. \( 2x + y = -4 \) and \( y + 2x = 3 \)
2. Without graphing, determine whether the following systems of linear equations will have one solution, no solution, or infinitely many solutions.
   a. \( x + y = 5 \) and \( x + y = 6 \)
   b. \(-3x + 9y = 15\) and \( y = \frac{1}{3}x + \frac{5}{3} \)
   c. \( y = 6 \) and \( y = 2x + 1 \)
   d. \( x - y = 5 \) and \( x + y = 5 \)

3. How many solutions does the system of linear equations graphed below have? How do you know?

4. One equation in a system of linear equations is \( y = x - 4 \).
   a. Write a second equation for the system so that the system has only one solution.
   b. Write a second equation for the system so that the system has no solution.
   c. Write a second equation for the system so that the system has infinite solutions.
5. The grid below shows the graph of a line and a parabola (the u-shaped graph).

a. How many solutions do you think there are to this system of equations? Explain your answer.

b. Estimate the solution(s) to this system of equations.

c. The following is the system of equations graphed above.

\[ y = x + 1 \]
\[ y = (x - 2)^2 + 1 \]

How can you verify whether the solution(s) you estimated in part b) are correct?

d. Verify the solution(s) from part b).
### 5.1d Self-Assessment: Section 5.1

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept.

<table>
<thead>
<tr>
<th>Skill/Concept</th>
<th>Beginning Understanding</th>
<th>Developing Skill and Understanding</th>
<th>Deep Understanding, Skill Mastery</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Solve simultaneous linear equations by graphing.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Understand what it means to solve a system of equations.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Identify and provide examples of systems of equations that have one solution, infinitely many solutions, or no solution.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Interpret the solution to a system in a context.</td>
<td></td>
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</tr>
</tbody>
</table>
Section 5.2: Solve Simultaneous Linear Equations Algebraically

Section Overview:
In this section, students are solving simultaneous linear equations that have one, no, or infinitely many solutions using algebraic methods. The section utilizes concrete models and real world problems in order to help students to grasp the concepts of substitution and elimination. Students then solve systems of linear equations abstractly by manipulating the equations. Students then apply the skills they have learned in order to solve real world problems that can be modeled and solved using simultaneous linear equations.

Concepts and Skills to Master:
By the end of this section, students should be able to:

1. Solve simultaneous linear equations algebraically.
2. Determine which method of solving a system of linear equations may be easier depending on the problem.
3. Create a system of equations to model a real world problem, solve the system, and interpret the solution in the context.
5.2a Class Activity: Introduction to Substitution

**Directions:** Find the value of each shape.

1. \[ \bigcirc + \bigcirc + \square = 25 \]
   \[ \square = 5 \]
   \[ \bigcirc = \ldots \]
   How did you determine the circle’s value?

2. \[ \bigcirc + \bigcirc + \bigcirc + \square = 18 \]
   \[ \bigcirc + \square = 10 \]
   \[ \square = \ldots \]
   \[ \bigcirc = \ldots \]
   How did you determine the value of each shape?

3. \[ \bigcirc + \bigcirc + \square + \square = 20 \]
   \[ \bigcirc + \bigcirc + \square = 17 \]
   \[ \square = \ldots \]
   \[ \bigcirc = \ldots \]
   How did you determine the value of each shape?

4. \[ \triangle + \triangle + \triangle = 27 \]
   \[ \triangle + \square = 8 \]
   \[ \triangle = \ldots \]
   \[ \square = \ldots \]
   How did you determine the value of each shape?
5. \[\begin{align*}
\star + \star + \triangle + \triangle &= 16 \\
\star + \star + \star + \triangle + \triangle &= 26
\end{align*}\]

How did you determine the value of each shape?

6. \[\begin{align*}
\star + \star + \bigcirc + \bigcirc + \bigcirc &= 19 \\
\star + \star + \bigcirc &= 13
\end{align*}\]

How did you determine the value of each shape?

7. \[\begin{align*}
\bigcirc + \square + \square &= 30 \\
\square &= \bigcirc + \bigcirc
\end{align*}\]

How did you determine the value of each shape?

8. \[\begin{align*}
\bigcirc + \bigcirc &= 16 \\
\triangle + \triangle + \triangle + 4 &= 16
\end{align*}\]

How did you determine the value of each shape?

9. \[\begin{align*}
\square + \square + \star &= 21 \\
\square + \square + \square + \square + \star + \star &= 42
\end{align*}\]
**Directions:** Draw a picture of each equation with shapes and then find the value of each shape.

10. $3x + 2y = 41$

   $2y = 8$

11. $2x + y = 9$

   $x + y = 5$

12. $x + 3y = 41$

   $x + 2y = 32$

13. $2x + 2y = 18$

   $2x = y$
**Challenge Questions:** Find the value of each variable using shapes.

14. \(x + 2y = 46\)
   \[y + 3z = 41\]
   \[3z = 27\]

15. \(2x + z = 46\)
   \[3z = 18\]
   \[2y + z = 40\]

16. \(2x + 2y = 50\)
   \[2x + y = 42\]
   \[y + 2z = 18\]
5.2a Homework: Introduction to Substitution

Directions: Find the value of each shape. Explain how you determined each.

1. 
\[
\begin{align*}
\bigcirc + \bigcirc + \bigcirc + \bigcirc + \square + \square &= 34 \\
\bigcirc + \bigcirc + \square + \square &= 26
\end{align*}
\]
\[
\begin{align*}
\square &= \_\_\_\_ \\
\bigcirc &= \_\_\_\_
\end{align*}
\]

2. 
\[
\begin{align*}
\triangle + \triangle + \triangle + \hexagon &= 27 \\
\triangle + \triangle + \hexagon &= 20
\end{align*}
\]
\[
\begin{align*}
\hexagon &= \_\_\_\_ \\
\triangle &= \_\_\_\_
\end{align*}
\]

3. 
\[
\begin{align*}
\star + \star + \star + \star + \star + \star + \moon + \moon + \moon + \moon &= 42 \\
\moon + \moon &= \star + \star + \star + \star + \star
\end{align*}
\]
\[
\begin{align*}
\star &= \_\_\_\_ \\
\moon &= \_\_\_\_
\end{align*}
\]

4. 
\[
\begin{align*}
\drop + \drop + \drop + \drop + \drop + \drop &= \cloud \\
\drop + \drop + \drop + \drop + 8 &= \cloud
\end{align*}
\]
\[
\begin{align*}
\cloud &= \_\_\_\_ \\
\drop &= \_\_\_\_
\end{align*}
\]
Directions: Draw a picture of each equation with shapes and then find the value of each shape.

5. \( x + y = 15 \)
   
   \( y = x + 10 \)

6. \( y + x = 5 \)
   
   \( x = y - 3 \)

7. \( y = 4x \)
   
   \( x + y = 5 \)

8. \( 2x + y = 7 \)
   
   \( x + y = 1 \)
9. \[3x + 4y = 19\]

\[3x + 6y = 33\]

10. \[5x + 6y = 100\]

\[4x + 6y = 92\]
5.2b Class Activity: Substitution Method for Solving Systems of Equations

Directions: Write a system of equations from the shapes. Find the value of each shape.

1.

\[
\text{□} + \text{□} + \text{□} = 30
\]

\[
\text{□} = \text{□} + \text{□}
\]

System of Equations: How did you determine the value of each shape?

2.

\[
\text{△} + \text{△} + \text{△} + \text{□} = 12
\]

\[
\text{△} + \text{△} + \text{△} + \text{△} + \text{□} + \text{□} = 26
\]

System of Equations: How did you determine the value of each shape?

To solve any system of linear equations using substitution, do the following:

1. Rewrite one of the equations so that one variable is expressed in terms of the other (solve one of the equations for one of its variables).
2. Substitute the expression from step 1 into the other equation and solve for the remaining variable.
3. Substitute the value from step 2 into the equation from step 1 and solve for the remaining variable.
4. Check the solution in each of the original equations.

Revisit problem #2 from above and use these steps to solve.
3. \[\begin{align*}
\square + \square + 7 &= \bigcirc \\
\square + \square + \square + \bigcirc + \bigcirc &= 35
\end{align*}\]

   a. Write a system of equations for the picture above.

   b. Solve this system of equations using substitution showing all steps. Check your solution.

4. \[\begin{align*}
\star + \star + \star + 1 &= \triangle \\
\star + \star + 3 &= \triangle
\end{align*}\]

   a. Write a system of equations for the picture above.

   b. Solve this system of equations using substitution showing all steps. Check your solution.
5.

\[
\begin{align*}
\text{\(\bigcirc\)} + \text{\(\bigcirc\)} + \text{\(\bigcirc\)} + 5 &= \text{\(\triangle\)} \\
\text{\(\bigcirc\)} + \text{\(\bigcirc\)} + \text{\(\bigcirc\)} + 3 &= \text{\(\triangle\)} \\
\end{align*}
\]

a. Write a system of equations for the pictures above.

b. Solve this system of equations using substitution showing all steps. Check your solution.

c. Describe what you would see in a graph of this system.

6.

\[
\begin{align*}
\text{\(\square\)} + \text{\(\square\)} + \text{\(\star\)} &= 21 \\
\text{\(\square\)} + \text{\(\square\)} + \text{\(\square\)} + \text{\(\square\)} + \text{\(\star\)} + \text{\(\star\)} &= 42 \\
\end{align*}
\]

a. Write a system of equations for the pictures above.

b. Solve this system of equations using substitution showing all steps. Check your solution.

c. Describe what you would see in a graph of this system.
**Directions:** Solve each system using the substitution method.

<table>
<thead>
<tr>
<th>System 1</th>
<th>System 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>7. ( y = 5x + 4 ) ( y = -3x - 12 )</td>
<td>8. ( y = 6x + 4 ) ( y = 6x - 10 )</td>
</tr>
<tr>
<td>9. ( y = x + 2 ) ( x + 3y = -2 )</td>
<td>10. ( x = 2y - 4 ) ( x + y = 2 )</td>
</tr>
<tr>
<td>11. ( y - x = 5 ) ( 2x + y = -10 )</td>
<td>12. ( 2x + y = 5 ) ( y = -5 - 2x )</td>
</tr>
<tr>
<td>Equation 1</td>
<td>Equation 2</td>
</tr>
<tr>
<td>------------</td>
<td>------------</td>
</tr>
</tbody>
</table>
| 13. $y + x = 5$  
  $x = y - 3$ | 14. $x + y = 4$  
  $y = -x + 4$ |
| 15. $x + 2y = 7$  
  $2x + 3y = 12$ | 16. $6x + y = -5$  
  $-12x - 2y = 10$ |
Directions: The following are examples of real-world problems that can be modeled and solved with systems of linear equations. Answer the questions for each problem.

17. Nettie’s Fine Clothing is having a huge sale. All shirts are $3 each and all pants are $5 each. You go to the sale and buy twice as many pants as shirts and spend $66.

The following system of equations models this situation where \( s = \) number of shirts and \( p = \) number of pants:
\[
\begin{align*}
\text{s} &= 2p \\
3s + 5p &= 66
\end{align*}
\]

a. Write in words what each of the equations in the system represents in the context.
\[
\begin{align*}
\text{s} &= 2p & \text{number of shirts is twice the number of pants} \\
3s + 5p &= 66 & \text{total spent on shirts and pants}
\end{align*}
\]

b. Solve this system using substitution to determine how many of each item you bought. Write your answer in a complete sentence.

18. Xavier and Carlos have a bet to see who can get more “friends” on a social media site after 1 month. Carlos has 5 more friends than Xavier when they start the competition. After much work, Carlos doubles his amount of friends and Xavier triples his. In the end they have a total of 160 friends together.

The following system of equations models this situation where \( c = \) the number of friends Carlos starts with and \( x = \) the number of friends Xavier starts with.
\[
\begin{align*}
c &= x + 5 \\
2c + 3x &= 160
\end{align*}
\]

a. Write in words what each of the equations in the system represents in the context.
\[
\begin{align*}
c &= x + 5 & \text{Carlos has 5 more friends than Xavier at start} \\
2c + 3x &= 160 & \text{total friends after 1 month}
\end{align*}
\]

b. Solve this system using substitution to determine how many friends each boy started with. Write your answer in a complete sentence.
5.2b Homework: Substitution Method for Solving Systems of Equations

**Directions:** Solve each system of linear equations using substitution.

<table>
<thead>
<tr>
<th></th>
<th>1. ( y = 4x )</th>
<th>2. ( x = -4y )</th>
<th>3. ( y = x - 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( x + y = 5 )</td>
<td>( 3x + 2y = 20 )</td>
<td>( x + y = 3 )</td>
</tr>
<tr>
<td>4.</td>
<td>( 3x - y = 4 )</td>
<td>( x + y = 3 )</td>
<td>( y = -3x + 6 )</td>
</tr>
<tr>
<td></td>
<td>( 2x - 3y = -9 )</td>
<td>( x + 2y = 3 )</td>
<td>( 9x + 3y = 18 )</td>
</tr>
<tr>
<td>7.</td>
<td>( 2x + y = -15 )</td>
<td>( x - y = 6 )</td>
<td>( x - y = 10 )</td>
</tr>
<tr>
<td></td>
<td>( y - 5x = 6 )</td>
<td>( 2x = 12 + 2y )</td>
<td>( 3x - 3y = 30 )</td>
</tr>
<tr>
<td>10.</td>
<td>( y = 4x - 2 )</td>
<td>( y = 3x + 4 )</td>
<td>( 2x + 2y = 6 )</td>
</tr>
<tr>
<td></td>
<td>( 3x + y = 5 )</td>
<td>( y = x - 7 )</td>
<td>( x + y = 0 )</td>
</tr>
</tbody>
</table>
5.2c Class Activity: Elimination Method for Solving Systems of Linear Equations

1. Ariana and Emily are both standing in line at Papa Joe’s Pizza. Ariana orders 4 large cheese pizzas and 1 order of breadsticks. Her total before tax is $34.46. Emily orders 4 large cheese pizzas and 2 orders of breadsticks. Her total before tax is $36.96. Determine the cost of 1 large cheese pizza and 1 order of breadsticks. **Explain** the method you used for solving this problem.

2. Carter and Sani each have the same number of marbles. Sani’s little sister comes in and takes some of Carter’s marbles and gives them to Sani. After she has done this, Sani has 18 marbles and Carter has 10 marbles. How many marbles did each of the boys start with? How many marbles did Sani’s sister take from Carter and give to Sani?
3.

\[ \begin{array}{c}
\text{○} + \text{○} + \text{○} + \text{○} + \text{□} = 18 \\
\text{○} + \text{○} + \text{□} = 10 \\
\end{array} \]

\[ \begin{array}{c}
\text{○} = \_ \\
\text{□} = \_ \\
\end{array} \]

a. Find the value of each shape.

b. Which of the problems from the previous page is this similar to? Compare the strategies you used to solve these problems.

4.

\[ \begin{array}{c}
\text{○} + \text{□} = 15 \\
\text{○} - \text{□} = 7 \\
\end{array} \]

\[ \begin{array}{c}
\text{○} = \_ \\
\end{array} \]

a. Find the value of each shape.

b. Which of the problems from the previous page is this similar to? Compare the strategies you used to solve these problems.
5.2d Class Activity: Elimination Method of Solving Linear Systems

**Directions:** Write a system of equations from the shapes. Find the value of each shape. Show the solving actions algebraically.

1. 
   \[ \bigcirc + \bigcirc + \square = 14 \]
   \[ \bigcirc + \bigcirc - \square = 10 \]
   \[ \bigcirc = \_\_\_ \]
   \[ \square = \_\_\_ \]

2. 
   \[ \square + \bigcirc + \bigcirc = 19 \]
   \[ \bigcirc - \square = 11 \]
   \[ \bigcirc = \_\_\_ \]
   \[ \square = \_\_\_ \]

3. 
   \[ \square + \square + \bigcirc = 27 \]
   \[ \bigcirc - \square - \square = 15 \]
   \[ \bigcirc = \_\_\_ \]
   \[ \square = \_\_\_ \]
4. The name of the method you are using to solve the systems of linear equations above is **elimination**. Why do you think this method is called **elimination**?

**Directions:** Write a system of equations from the shapes. Find the value of each shape. Show the solving actions algebraically.

5. 

\[
\begin{align*}
\star + \star - \bigcirc &= 8 \\
\star + \star - \bigcirc - \bigcirc &= 4
\end{align*}
\]

\[
\star = \_\_\_ \\
\bigcirc = \_\_\_
\]

6. 

\[
\begin{align*}
\triangle + \triangle + \triangle + \triangle + \star + \star &= 8 \\
\triangle + \triangle + \star + \star &= -6
\end{align*}
\]

\[
\triangle = \_\_\_ \\
\star = \_\_\_
\]

7. How are problems 5 and 6 different from #1 – 3. Describe in your own words how you solved the problems in this lesson.
**Directions:** Solve each system of linear equations using **elimination.** Make sure the equations are in the same form first.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
</table>
| 8. | $x + y = -3$
    | $2x - y = -3$ |   |
| 9. | $x + y = 50$
    | $-x - y = -50$ |   |
| 10. | $x + y = 3$
    | $2x - y = -3$ |   |
| 11. | $2x - y = -3$
     | $3x - y = 1$ |   |
| 12. | $2x - y = 9$
     | $x + y = 3$ |   |
| 13. | $x = y + 3$
     | $x - 2y = 3$ |   |
| 14. | $2x + y = 6$
     | $2x + y = -7$ |   |
| 15. | $3x - y = 1$
     | $x = -y + 3$ |   |
| 16. | $7x - 4y = -30$
     | $3x + 4y = 10$ |   |
5.2d Homework: Elimination Method of Solving Linear Systems

**Directions:** Solve each system of linear equations using **elimination**. Make sure the equations are in the same form first.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $6x - y = 5$</td>
<td>2. $x + 4y = 9$</td>
<td>3. $x + 5y = -8$</td>
</tr>
<tr>
<td></td>
<td>$3x + y = 4$</td>
<td>$-x - 2y = 3$</td>
</tr>
<tr>
<td>4. $2x + y = 7$</td>
<td>5. $4x + 3y = 18$</td>
<td>6. $-5x + 2y = 22$</td>
</tr>
<tr>
<td></td>
<td>$x + y = 1$</td>
<td>$4x = 8 + 2y$</td>
</tr>
<tr>
<td>7. $6x - 3y = 36$</td>
<td>8. $-4x + y = -12$</td>
<td>9. $x + y = 7$</td>
</tr>
<tr>
<td></td>
<td>$5x = 3y + 30$</td>
<td>$-y + 6x = 8$</td>
</tr>
</tbody>
</table>
5.2e Class Activity: Elimination Method Multiply First

1. Solve the following system of linear equations using elimination:

\[
\begin{align*}
4x + y &= 7 \\
-2x - 3y &= -1
\end{align*}
\]

To solve any system of linear equations using elimination, do the following:

1. Write both equations in the same form.
2. Multiply the equations by nonzero numbers so that one of the variables will be eliminated if you take the sum or difference of the equations.
3. Take the sum or difference of the equations to obtain a new equation in just one unknown.
4. Solve for the remaining variable.
5. Substitute the value from step 4 back into one of the original equations to solve for the other unknown.
6. Check the solution in each of the original equations.

**Directions:** Solve each system of linear equations using elimination.

<table>
<thead>
<tr>
<th>2. ( x + 2y = 15 ) &amp; 3. ( -3x + 2y = -8 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 5x + y = 21 ) &amp; ( 6x - 4y = -20 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>4. ( 2x - 5y = 1 ) &amp; 5. ( 3x - 2y = 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( -3x + 7y = 3 ) &amp; ( 5x - 5y = 10 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>6. ( 9x + 13y = 10 ) &amp; 7. ( -16x + 2y = -2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( -9x - 13y = 8 ) &amp; ( y = 8x - 1 )</td>
</tr>
</tbody>
</table>
Directions: Write a system of equations for each word problem below and then solve the system of equations using elimination. Write your answer in a complete sentence.

8. The student officers are buying packs of streamers and balloons to decorate for a school dance. Packs of balloons cost $3.50 and packs of streamers cost $2. If the student officers have a total of 12 packs of decorations and spent $31.50, how many packs of balloons did they buy? How many packs of streamers did they buy? Write the solution in a complete sentence.

Define your Variables:

Equation for Number of Packs of Decorations:

Equation for Cost of Decorations:

Solve:

Solution (in a complete sentence):

9. Jayda has a coin collection consisting of nickels and dimes. She has 28 coins worth $2.25. How many of each coin does she have? Write the solution in a complete sentence.

Define your Variables:

Equation for Number of Coins:

Equation for Value of Coins:

Solve:

Solution (in a complete sentence):
### 5.2e Homework: Elimination Method Multiply First

**Directions:** Solve each system using elimination.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
</table>
| 1. | $x + y = 3$
    | $-2x + 4y = 6$
| 2. | $4x + y = -8$
    | $3x + 3y = 3$
| 3. | $2x + y = 7$
    | $4x + 2y = 14$
| 4. | $2x + 3y = -10$
    | $-4x + 5y = -2$
| 5. | $x + 3y = 1$
    | $-5x + 4y = -24$
| 6. | $-3x - y = -15$
    | $8x + 4y = 48$
| 7. | $3x - y = 10$
    | $2x + 5y = 35$
| 8. | $x + y = 15$
    | $-2x - 2y = 30$
Directions: Write a system of equations for each word problem below and then solve the system of equations using elimination. Write your answer in a complete sentence.

9. Tickets for a matinee are $5 for children and $8 for adults. The theater sold a total of 142 tickets one day. Ticket sales were $890. How many of each type of ticket did the theater sell? Write the solution in a complete sentence.

Define your Variables:

Equation for Number of Tickets Sold:

Equation for Ticket Sales:

Solve:

Solution (in a complete sentence):

10. Jasper has a coin collection consisting of quarters and dimes. He has 50 coins worth $8.60. How many of each coin does he have? Write the solution in a complete sentence.

Define your Variables:

Equation for Number of Coins:

Equation for Value of Coins:

Solve:

Solution (in a complete sentence):
Directions: Solve each system of linear equations by choosing the method that you feel is easiest for a given problem (graphing, substitution, or elimination).

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>(2x - 3y = 12)</td>
<td>(x = 4y + 1)</td>
</tr>
<tr>
<td>2.</td>
<td>(x + y = 3)</td>
<td>(3x - 4y = -19)</td>
</tr>
<tr>
<td>3.</td>
<td>(y = x - 6)</td>
<td>(y = x + 2)</td>
</tr>
<tr>
<td>4.</td>
<td>(y - 2x = 1)</td>
<td>(2x + y = 5)</td>
</tr>
<tr>
<td>5.</td>
<td>(x - y = 2)</td>
<td>(4x - 3y = 11)</td>
</tr>
<tr>
<td>6.</td>
<td>(x - y = 0)</td>
<td>(2x + 4y = 18)</td>
</tr>
<tr>
<td>7.</td>
<td>(3y - 9x = 1)</td>
<td>(y = 3x + \frac{1}{3})</td>
</tr>
<tr>
<td>8.</td>
<td>(x + 2y = 6)</td>
<td>(-7x + 3y = -8)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td></td>
</tr>
</tbody>
</table>
| 9. | \( y = 2 \)  
    | \( y = 3x + 2 \)  
| 10. | \( 3x = y - 20 \)  
     | \( -7x + y = 40 \)  
| 11. | \( 3x + 2y = -5 \)  
    | \( x - y = 10 \)  
| 12. | \( 2x - 5y = 6 \)  
    | \( 2x + 3y = -2 \)  
| 13. | \( y = 4x - 3 \)  
    | \( y = x + 6 \)  
| 14. | \( y = x + 5 \)  
    | \( y = 2x - 10 \)  
| 15. | \( y = -x + 5 \)  
    | \( x - 4y = 10 \)  
| 16. | \( 2y = x - 5 \)  
    | \( 2y = x + 5 \)  

### 5.2f Homework: Solving Systems of Equations Mixed Strategies

**Directions:** Solve each system of linear equations by choosing the method that you feel is easiest for a given problem (graphing, substitution, or elimination).

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
</table>
| 1. | \( y = 4x \)  
\( x + y = 5 \) | 2. | \( x = -4y \)  
\( 3x + 2y = 20 \) |
| 3. | \( y = x - 1 \)  
\( y = -x + 3 \) | 4. | \( 3x - y = 4 \)  
\( 2x - 3y = -9 \) |
| 5. | \( x + 5y = 4 \)  
\( 3x + y = -2 \) | 6. | \( y = -x + 10 \)  
\( y = 10 - x \) |
| 7. | \( y = 2x \)  
\( x + y = 12 \) | 8. | \( y = 2x - 5 \)  
\( 4x - y = 7 \) |
5.2g Self-Assessment: Section 3.2

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept.

<table>
<thead>
<tr>
<th>Skill/Concept</th>
<th>Beginning Understanding</th>
<th>Developing Skill and Understanding</th>
<th>Deep Understanding, Skill Mastery</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Solve simultaneous linear equations algebraically.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Determine which method of solving a system of linear equations may be easier depending on the problem.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Create a system of equations to model a real world problem, solve the system, and interpret the solution in the context.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Section 5.3: Solve Real World Problems Using Simultaneous Linear Equations

Section Overview:
Students will apply the skills they have learned in the previous sections in order to solve real world problems that can be modeled and solved using simultaneous linear equations. Students will create systems of linear equations and solve them, deciphering when one solving strategy may be easier. They will also be required to interpret the solution in the context and make decisions based on the solution. It is encouraged that by the end of this section, students are using graphing calculators in order to solve applications where appropriate.

Concepts and Skills to Master:
By the end of this section, students should be able to:

1. Create systems of linear equations to model real world problems.
2. Solve real world problems using systems of linear equations using a variety of strategies.
3. Interpret the solution to a system of linear equations in a context and make decisions based on the solution.
5.3a Class Activity: Revisiting Chickens and Pigs

1. A farmer saw some chickens and pigs in a field. He counted 60 heads and 176 legs. Determine exactly how many chickens and pigs he saw.
   a. Solve the problem using the methods/strategies studied in this chapter. Solve in as many different ways as you can and make connections between the strategies.

b. Which method do you prefer using to solve this problem? Use your preferred method to determine the number of chickens and pigs in a field with 45 heads and 146 legs.
Directions: Solve each of the following problems by writing and solving a system of equations. Use any method you wish to solve. Write your answer in a complete sentence.

2. A restaurant needs to order stools and chairs. Each stool has 3 legs and each chair has 4 legs. The manager wants to be able to seat 36 people. The restaurant has hard wood floors and doesn’t want to scratch them. Therefore, they have ordered 129 plastic feet covers for the bottom of the legs to ensure the stools and chairs don’t scratch the floor. How many chairs and how many stools did the restaurant order?

3. The admission fee at a local zoo is $1.50 for children and $4.00 for adults. On a certain day, 2,200 people enter the zoo and $5,050 is collected. How many children and how many adults attended?

4. In 1982, the US Mint changed the composition of pennies from all copper to zinc with copper coating. Pennies made prior to 1982 weigh 3.1 grams. Pennies made since 1982 weigh 2.5 grams. If you have a bag of 1,254 pennies, and the bag weighs 3,508.8 grams, how many pennies of each time period are there in the bag?
5.3a Homework: Applications of Linear Systems

Directions: Solve each of the following problems by writing and solving a system of equations. Use any method you wish to solve. Write your answers in a complete sentence.

1. The department store down the street is having a sale on shirts and pants. All shirts are $5 and all pants are $12. Jennifer buys 18 items at a total cost of $160.

2. Tickets to the local basketball arena cost $54 for lower bowl seats and $20 for upper bowl seats. A large group purchased 123 tickets at a cost of $4,262. How many of each type of ticket did they purchase?

3. Sarah has $400 in her savings account and she has to pay $15 each month to her parents for her cell phone. Darius has $50 and he saves $20 each month from his job walking dogs for his neighbor. At this rate, when will Sarah and Darius have the same amount of money? How much money will they each have?

4. An exam worth 145 points contains 50 questions. Some of the questions are worth two points and some are worth five points. How many two-point questions are on the test? How many five-point questions are on the test?
## 5.3b Self-Assessment: Section 5.3

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept.

<table>
<thead>
<tr>
<th>Skill/Concept</th>
<th>Beginning Understanding</th>
<th>Developing Skill and Understanding</th>
<th>Deep Understanding, Skill Mastery</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Create systems of linear equations to model real world problems.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Solve real world problems using systems of linear equations using a variety of strategies.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Interpret the solution to a system of linear equations in a context and make decisions based on the solution.</td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>