Table of Contents

CHAPTER 8: GEOMETRY PART 2: MEASUREMENT IN 2-3 DIMENSIONS, PLANE - SECTIONS OF SOLIDS (2 WEEKS) ......................................................................................................................................................................................... 32

8.0 Anchor Problem: ........................................................................................................................................................................ 36

SECTION 8.1: MEASUREMENT IN TWO DIMENSIONS ................................................................................................................................. 37

8.1a Class Activity: Differentiate Area and Perimeter ................................................................................................................................. 38
8.1a Homework: Perimeter and Area Problem Solving ............................................................................................................................... 42
8.1b Class Activity: Areas of Irregular Shapes ........................................................................................................................................... 43
8.1b Homework: Areas of Irregular Shapes ........................................................................................................................................... 45
8.1c Class Activity: Areas of Irregular Shapes and Expressions ............................................................................................................... 47
8.1c Homework: Areas of Irregular Shapes and Expressions ............................................................................................................... 51
8.1d Class Activity: Review Areas of Triangles, Parallelograms, Trapezoids; Circle Area and Circumference .................................. 55
8.1d Homework: Review Areas of Triangles, Parallelograms, Trapezoids; Circle Area and Circumference .................................. 58
8.1f Self-Assessment: Section 8.1 ................................................................................................................................................................. 60

SECTION 8.2: 2D PLANE SECTIONS FROM 3D FIGURES AND 3D MEASUREMENT ............................................................................................. 61

8.2a Class Activity: 2D Plane Sections of Cubes and Prisms (play-dough & dental floss) ............................................................................... 62
8.2a Homework: 3D Objects ................................................................................................................................................................. 67
8.2b Class Activity: 2D Plane Sections on Cylinders and More ......................................................................................................................... 68
8.2b Homework: Area of Plane Sections ............................................................................................................................................... 72
8.2c Class Activity: Nets of 3D Objects .................................................................................................................................................. 73
8.2c Homework: Nets of 3D Objects .................................................................................................................................................. 75
8.2c Additional Practice: Surface Area and Volume .................................................................................................................................. 76
8.2d Class Activity: Growing and Shrinking ........................................................................................................................................... 77
8.2d Homework: Growing and Shrinking Stuff .................................................................................................................................. 78
8.2e Project: Packing Packages ............................................................................................................................................................... 80
8.2f Self-Assessment: Section 8.2 ............................................................................................................................................................... 81

Play Dough Recipe......................................................................................................................................................................................... 82
Chapter 8: Geometry Part 2: Measurement in 2-3 Dimensions, Plane - Sections of Solids (2 weeks)

UTAH CORE Standard(s)
Draw construct, and describe geometrical figures and describe the relationships between them.
1. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. 7.G.A. 3

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.
2. Know the formulas for the area and circumference of a circle and use them to solve problems. 7.G.B.4
3. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. 7.G.B.6

Chapter Summary:
Throughout this chapter students develop and explore ideas in geometry around measures in one- two- and three-dimensions. Additionally, students will tie concepts learned previously in 6th and 7th grade to content in this chapter. Section 1 begins with a brief review and practice with perimeter and area of polygonal figures. Students then use their knowledge of area and perimeter to solve real-world problems, find the area of non-standard shapes and review ideas around percent increase/decrease and scale factor, and connect their understanding of one and two dimensional measures to adding and multiplying algebraic expressions by finding the perimeter and area of figures with variable expression side lengths.

Section 2 starts with students exploring plane sections of 3-D solids. Exercises in this section are designed to help students develop an intuitive understanding of dimension (building up and cutting down.) Students will notice that plane sections of 3-D objects are affected by a) the type of solid with which one starts, e.g right prism versus a pyramid and b) the angle of the cut to the base and/or other faces. This observation should help students understand when and why parallel (and/or perpendicular) cuts to a specific face are needed to create uniform cross sections and then by extension, uniform cubed units for finding volume. In other words, the study of plane sections here is to help students develop an understanding of the structure of a solid and procedures for finding volumes. Throughout the exercises, students should develop a stronger understanding of units of measure for one-, two-, and three-dimensions. Towards the end of this section student review the use of nets (a concept from 6th grade) to find surface area of prisms and cylinders and then to differentiate this measure from volume, which they will also find.

VOCABULARY: face, edges, vertices, symmetry, axis of symmetry, equidistant, prism, cylinder, cross-section, perimeter, area, polygon, rectangle, square, triangle, trapezoid, quadrilateral, circle, circumference, polyhedron, cube, pyramid, volume, surface area

CONNECTIONS TO CONTENT:
Prior Knowledge: In 6th grade students found areas of special quadrilaterals and triangles. They also found areas of other objects by decomposing them into rectangles and triangles. In Chapter 5 of this text, students extended that understanding to finding the area of a circle. Also in 6th grade, students found volume of rectangular prisms and their surface areas by using nets. During their study of geometry in 6th grade, students should have learned that the height and base of an object are always perpendicular to each other. They will build on this understanding as they apply their knowledge of area and volume to real life contexts and as they explore cross sections.
Future Knowledge: In 8th grade, students will continue working with volume, formalizing algorithms for volume of cylinders and adding volume of cones and spheres. Students explore cross sections of objects in 7th grade to understand how dimensions are related to each other and to the algorithms for surface area and volume. Further, ideas developed through cross section activities are foundation in the study of calculus. Lastly, ideas about planar sections will be extended in secondary mathematics when students explore Cavalieri’s Principle.
MATHEMATICAL PRACTICE STANDARDS (emphasized):

<table>
<thead>
<tr>
<th>Make sense of problems and persevere in solving them.</th>
<th>Students will be given a variety of contextual problems with which they will need to make sense and persevere. For example: 8.0 Anchor Problem; The city is looking to build a new swimming pool in city park. In their city council meeting, they have determined that they want the pool to hold no more than 2500 m$^3$ of water, or it will cost too much to keep it filled. Help the city council to choose a design for the swimming pool. Design 3 different swimming pools that will each hold at least 2000 m$^3$ but no more than 2500 m$^3$ of water.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reason abstractly and quantitatively.</td>
<td>Students will use their understanding of area and volume to reason in a variety of contexts: 8.1e Class Activity; Write an expression to find the area. If possible, find the exact area.</td>
</tr>
<tr>
<td>Construct viable arguments and critique the reasoning of others.</td>
<td>Students will apply their understanding of perimeter and area to construct and critique arguments: 8.1g Class Activity; Mike, Juliana and Joe were working together to make a garden larger. Mike said, “We have to buy more fencing because if we increase the area of the garden we will need more fencing to go around.” Juliana had a different opinion. “That’s not true,” she said, “We can use the same amount of fencing and move it to make the area of the garden larger.” Joe disagreed with both Mike and Juliana. He said, “I know a way that we can make the garden larger and use less fencing. Who is right?</td>
</tr>
<tr>
<td>Model with Mathematics.</td>
<td>Students will use models to explore concepts in geometry such are using play-doe and string to create cross sections of prisms. Throughout the chapter they will connect models to algorithms.</td>
</tr>
<tr>
<td>Attend to Precision</td>
<td>Careful attentions should be paid to explanations and units throughout this chapter. Students will be expected to attend to several ideas at the same time. Students should attend to precision as they explain ideas throughout this chapter. For example, when discussing cross sections, students should use precise language in describing the angle of cuts to the base, faces and/or edges.</td>
</tr>
<tr>
<td>Look for and make use of structure</td>
<td>Students will connect ideas of one-, two-, and three-dimensional measures to simplifying numeric and algebraic expressions, for example: 8.2 d Homework, Determine which expression(s) will give the surface area or volume for a 3-D object. a. $2(6+2+3)$ b. $3 \cdot 2 \cdot 6$ c. $(2 \cdot 6 + 2 \cdot 3 + 3 \cdot 6)2$ d. $2 \cdot 3 \cdot 2 + 2 \cdot 3 \cdot 6 + 2 \cdot 6 + 2 \cdot 4$</td>
</tr>
</tbody>
</table>

34
<table>
<thead>
<tr>
<th>Use appropriate tools strategically.</th>
<th>Students will use a variety of tools in this chapter including play dough, rulers, graph paper, and calculators. Encourage students to make sense of ideas with tools.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Look for and express regularity in repeated reasoning</td>
<td>Students will note in this chapter that length is one-dimensional, area is two-dimensional and requires 2 length (2 one-dimensional measures) that are perpendicular, and volume is three dimensional and requires 3 lengths that are each perpendicular to each other.</td>
</tr>
</tbody>
</table>
8.0 Anchor Problem:

The city is looking to build a new swimming pool in city park. In their city council meeting, they determined that they want the pool to hold no more than $2500 \text{ m}^3$ of water in order to conserve water.

A. Help the city council to choose a design for the swimming pool. Design three different swimming pools that each hold at least $2000 \text{ m}^3$ but no more than $2500 \text{ m}^3$ of water.

B. Choose your favorite of the three designs from part A. Find new measures for the pool so that a) it holds twice as much water as your original design and b) it remains the same overall shape (but not size) as the original.
Section 8.1: Measurement in Two Dimensions

Section Overview:

This section involves a review and extension of previously learned skills (from 6th grade) involving perimeter and area of plane figures and volumes of right solids. The section begins with a review of perimeter and area and then moves to finding area of irregular figures; first with numeric side lengths and then with variable expression side length. Additionally, students continue work on skills developed earlier in the year including percent increase/decrease, scale factor for perimeter and area, and simplifying algebraic expressions. The end of the section focuses on all these skill in various real-world and mathematical contexts.

Concepts and Skills to be Mastered (from standards)

1. Solve problems involving area and circumference of a circle.
2. Solve real-world and mathematical problems involving area and perimeter of two-dimensional objects composed of polygons and/or circles.
3. Find scale factors and/or percent increase/decrease in area/perimeter contexts.
4. Solidify
8.1a Class Activity: Differentiate Area and Perimeter

REVIEW:

1. Rectangle Riddle #1: Can you figure out the dimensions for each rectangle? Use the clues below to draw the four different rectangles described.
   - Rectangle A has a perimeter equal to that of Rectangle B.
   - Rectangle A is a square.
   - Rectangle B has an area of 24 square units. It is as close to a square as possible for that area if the side lengths are whole numbers.
   - Rectangle C has a perimeter equal to the measure of the area of Rectangle B. Rectangles C’s length is 3 times the width.
   - Rectangle D has two different odd integers as length and width; each is 1 unit greater than the length and width of Rectangle B.
Use words and models to explain the difference between perimeter and area.

2. Rectangle Riddle #2: Can you figure out the dimensions for each rectangle? Use the clues below to draw the four different rectangles described.

- Rectangle 1 has the number representing area equal to twice the perimeter. Rectangle 1 is 8 units wide.
- Rectangle 2 has the same number representing the area and the perimeter; the perimeter is 1/2 the perimeter of Rectangle 1.
- Rectangle 3 has the same length as Rectangle 1. The area of Rectangle 3 is equal to the difference between the areas of Rectangles 1 and 2.
- Rectangle 4 has an area 6 square units less than the area of Rectangle 3 and a width 2 units more than the width of Rectangle 2.

3. What helped you most to accomplish “Rectangle Riddle #2”?
For questions 4-6 use a piece of graph paper to answer each:

4. List as many rectangles as you can that have an area of 36?

5. DRAW two different parallelograms that have the same area. State their base and height.

6. DRAW two different triangles that have an area of 12. State their base and height.

7. Suppose you start with a rectangle that has a base length of 6 and height of 4. If you triple the length of the base, what do you have to do to the height to have a new rectangle of the same area?
Activity:

Mike, Juliana and Joe were working together to make a garden larger. Mike said, “We have to buy more fencing because if we increase the area of the garden we will need more fencing to go around.”

Juliana had a different opinion. “That’s not true,” she said, “We can use the same amount of fencing and move it to make the area of the garden larger.”

Joe disagreed with both Mike and Juliana. He said, “I know a way that we can make the garden larger and use less fencing.

Who is right?

A. Group response: Use graph paper and string to help you think about the problem.
   a. Decide who you think is right (Mike, Juliana or Joe.)
   b. Come up with a possible answer with your group.
   c. Prepare your group explanation and presentation.
   d. Be certain to justify your conclusion with reasoning and calculation.

B. Challenge: Examine the Garden Problem using a spreadsheet. Consider all possible rectangles with perimeters of 36 units. How does the area change as you change the configuration of the perimeter?
8.1a Homework: Perimeter and Area Problem Solving

Answer each of the questions below.

1. What is the least number of tiles you can add to the figure below to create a shape with a perimeter of 16?
   Note: When adding a tile, the new tile must share at least one side with the original shape; each tile is 1 unit by 1 unit.

2. Use the figure above to answer a-g. Draw your answers. Note, an answer of just “yes” or “no” is not sufficient. Use pictures or words to justify your answer.
   a. Can you add a tile to this figure to increase the perimeter by 1? If so, how?
   b. Can you add a tile to this figure to increase the perimeter by 2? If so, how?
   c. Can you add a tile to this figure to increase the perimeter by 3? If so, how?
   d. Can you add a tile to this figure so that the perimeter doesn’t change? If so, how?
   e. Can you make more than one shape with the same perimeter, but different areas? Show your ideas with grid paper.
   f. Can you make more than one shape with the same area, but different perimeters? Show your ideas with grid paper.
   g. If you pick any whole number between 12 and 24, can you make a shape with that perimeter? Show your ideas.
8.1b Class Activity: Areas of Irregular Shapes

The following shapes have been drawn on dot paper. The distance between each dot represents one unit. Use what you have learned about area to find the area of each shape (A-L).

Use your knowledge of area and the problems that you completed above to find the area of the following irregular shapes.
1 a. Johona is building a deck off the back of her house. To the right is a sketch of it. She will need to have a full concrete foundation below it. Find the surface area of the concrete foundation.

b. Suppose Johona wants to build her deck onto a concrete foundation that is 1.5 ft. thick and has the same surface area as the deck. How many cubic feet of concrete will she need?

c. How many cubic yards will she need?

2. a. Hugo is making the tile pattern shown. The tile is a quadrilateral with four circles of the same size inside. He will paint the circles blue and the remaining part of the tile yellow. Find the area of the portion of the tile that will be yellow.

b. Find the area of the diamond-shaped piece in the middle of the tile.

c. What portion (percent) of the tile will be yellow?
8.1b Homework: Areas of Irregular Shapes

1. The following shapes have been drawn on dot paper. The distance between each dot represents one unit. Use what you have learned about area to find the area of each shape (A-J).

Solve the following area problems.

2 a. A lap pool with a length of 40 ft. and a width of 15 ft. is surrounded by a 5-ft. wide deck. Find the area of the deck.

2 b. Draw a picture of the deck if the deck if it is extended 3 feet in every direction, then find the area of the new deck.
3. A rectangular field with two semi-circles at each of the shorter ends of the field measures 100 yards long and 40 yards wide. It is surrounded by a track that is 5 yards wide. Find the area of the field which includes the two semi-circles on each of the shorter ends of the rectangle. Find the area of the track.

4. Laura is painting a sign for the new Post Office. She will paint the triangular portion blue and the lower rectangular portion red. Find the area of sign that she will paint blue. What percent of the sign will be in blue?
8.1c Class Activity: Areas of Irregular Shapes and Expressions

Use your knowledge of area to find the area of the following shapes, if possible. Write an expression to show how you arrived at your answer.

1 a. Suppose you had a 60 x 100 meter plot of ground on which you were going to plant a garden. In the corner of your plot you want to build a 5 x 8 storage shed. How much area will be left for your garden? Does it matter which way the storage shed is oriented?

1 b. Suppose you have a 60 by 200 meter plot of ground on which you are going to plant a garden. You don’t want to plant the whole thing, just an area of 8000 m^2. If one side is 60 meters, how long will the other side (the 200 meter side) have to be to have an area of 8000m^2? What percent of the other side would you be using?

2 a.

2 b. Suppose you have a 60 by 200 meter plot of ground on which you are going to plant a garden. You don’t want to plant they whole thing, just an area of 8000 m^2. If one side is 60 meters, how long will the other side (the 200 meter side) have to be to have an area of 8000m^2? What percent of the other side would you be using?

What percent of your 60x200 meter plot of ground will be planted?
3. a

3 b.

3 c. Suppose you’re building a rectangular storage enclosure. The base of the enclosure is to be 60 by 100 meters. You need to construct a ramp into the enclosure as illustrated in 3 b. How long should the ramp be if you want the remaining area to be 5250 m$^2$?

4a. Find the area and perimeter:

4b. Find the area and perimeter:

Write an expression to represent the indicated measure for each of the following irregular shapes.
5. Find the area and perimeter for the figure below.

6. Find the area of the figure below.

7. Find the area and perimeter of the figure below:

8. Find the area of the figure below:
9. A bull’s eye is made of two concentric circles as shown below. The radius of the smaller circle is 4.5 inches. The larger circle has a radius of 9 inches. Use 3.14 to approximate the area of:
   a. The smaller circle
   b. The larger circle
   c. The space between the smaller circle and larger circle (orange area)
   d. What is the probability of hitting the green area?
   e. What is the probability of hitting the orange area?

10. Find the area of the orange part of the figure below.

11. The figure below shows a circle and a rectangle. The circle’s diameter is equal to the rectangle’s base. Find area of the shaded region and its perimeter. (use 3.14 for π)
8.1c Homework: Areas of Irregular Shapes and Expressions

Solve the following area problems. Write an expression showing how you got the area.

1. Find the area of the region.

2. Find the perimeter and area of the region.
   (use 3.14 for pi)

3. Jeremy needs to buy soil for the garden spot in his backyard. A sketch of the plot is to the right.
   a. Find the area of the garden.
   b. How many cubic feet of soil will he need to buy if he covers the area in 6 inches of soil?
   c. How many cubic feet of soil will he need to buy if he covers the area in one and a half feet of soil?
   d. How many feet of fencing will he need to enclose the garden if he fences the exact shape of the garden?
4. Nico is building a deck around the circular pool in his back yard. The pool has a radius of 15 ft. The deck will be 5 ft wide.
   a. Find the area of the deck.

5. a. A stage with a trapezoidal area upstage and a rectangular area downstage is illustrated i the figure to the right. Find the area of the stage.
   b. What is the area of just the rectangular portion of the stage?
   c. What portion of the stage is the rectangular portion?

6. Rachel is painting a sign for the new Health Center. Find the area of sign that she will need to paint red if she paints the whole background red.

Suppose Rachel decides to paint the lower portion (the 3 x 1 bottom portion) of the sign blue. What percent of the sign would be blue?
Write an expression to represent the **white area** of the following irregular shapes.

7. Find the perimeter of this region:

8. The circle has a radius of 2y.
Activity: In pairs, answer each of the following. Use a model to justify your answer.

1. Look back at question 3 from your homework (8.1c). How much more soil will Jeremy need to buy if he decided to cover his garden in 1.5 feet of soil rather than six inches of soil? Give your answer as a scale factor (twice as much? Three times as much? 1.5 times as much?) Explain:

2. Kara and Sharice are in a quilting competition. Both are stitching rectangular-shaped quilts. So far Kara’s has an area of 2,278 square inches with a height of 44 inches. Sharice’s quilt has an area of 2,276 square inches with a height of 47 inches. Whose quilt is wider? By how many inches is it wider?

3. Rufina bought two 12 foot pieces of lumber and two 8 foot pieces of lumber to create a border for her garden in her yard. She wants to use all the wood to enclose her garden. How should she use the four pieces to create a garden of largest possible area? What is the biggest area she can get with her four pieces of wood?

4. A triangle has an area of 390 square centimeters.
   a. If its height is 15 cm, what is the length of the base?
   b. Draw a triangle with an area of 390 square centimeters, height of 15 and base you found. Is there more than one triangle you can draw with those dimensions?
   c. Draw a triangle that has an area that is 150% of the original 390 cm² triangle. What are the dimensions of your larger triangle?
5. Julius drew a trapezoid that had bases of 15 and 11 inches and a height of 4 inches.
   a. What is the area of the trapezoid Julius drew?
   b. Can you draw a trapezoid that has the same area, but different dimensions?
   c. Draw a trapezoid with the same height but with bases that are a 5/2 scale factor of the original trapezoid.
   d. What is the area of the new trapezoid?

Review:
Draw a model and explain how to find the circumference of a circle:

Draw a model and explain how to find the area of a circle:

6. You’re making a 12 inch diameter pizza. You want the sauce to cover the pizza with a 1.5 inch ring left around the outside without sauce. (Use 3.14 for pi.)
   a. What is the area that the sauce will cover?

   b. What percent of the dough will be covered be sauce?

   c. If one 8 oz can of tomato sauce covers about 125 sq. inches of pizza dough, how many cans of sauce will you need to buy?
7. A circular swimming pool with a diameter of 32 feet is located exactly in the middle of a 40x40 square lot. For safety reasons the lot needs to have an 8’ fence on the perimeter of the entire lot.
   a. How long will the fence need to be?
   b. If the fence was around only the circular pool, how long would the fence be?
   c. Explain how much longer a fence around the whole yard is than a fence only around the pool using percent increase.
   d. What percent of the yard area does the pool take up?
   e. What is the area of the yard NOT taken up by the pool?

8. Look back at question 4 from the homework (8.1c). Nico’s pool currently takes up approximately 706.5 ft² of space in his yard (using 3.14 as an approximation for pi.) If Nico adds the five foot deck around the pool, what percent increase of space will this be?
8.1d Homework: Review Areas of Triangles, Parallelograms, Trapezoids; Circle Area and Circumference

1. Wallpaper comes in rolls that are 60 feet long and 2 feet wide. How many rolls of wallpaper will it take to cover 700 square feet?

2. A rectangular garden has an area of 45 square feet. One of the sides is 6 feet.
   a. What is the other side? 7
   b. You want to put a fence around it. How long will the fence need to be?
   c. You decide you want to increase the length of each side by a scale factor or 3.2. What are the new dimensions of your garden?
   d. What is the area of your new garden?

3. Mrs. Garcia has a table shaped like a regular trapezoid in her third grade classroom. The two parallel sides have lengths of 6 feet and 8 feet. The distance between them is 4 feet.
   a. What is the area of the top of Mrs. Garcia’s table?
   b. Suppose Mrs. Garcia has a 1.75 ft by 0.8 ft puzzle on the table. How much surface area is now available on her table?

4. The diameter of the earth is about 7926 miles.
   a. Find the distance around the earth at the equator.

   b. If there are 5280 feet in every mile, what is the distance around the Earth in feet?

   c. Suppose you can walk at a rate of 2 mile every 15 minutes. At this rate, how long would it take you to walk around the Earth?
5. A 12 foot by 16 foot office is being sectioned off into two triangular areas so that desks can be placed in opposite corners. The diagonal of the office (from corner to corner) is 20 feet. The manager needs a dividing curtain to hang from the ceiling around one of the triangles.

a. How long does the curtain need to be?

b. The carpet in each section will be a different color. How many square feet of carpet will be needed to cover each triangular section?

6. The three-point line in basketball is approximately a semi-circle with a radius of 19 feet and 9 inches. The entire court is 50 feet by 94 feet. What is the area of the court that results in 3 points?

7. Your neighbor’s backyard lawn is shaped like a rectangle. The back fence is 38.2 feet. The side fence is 32.6 feet. He will pay you $0.04 per square foot for mowing and $0.11 per foot for trimming all the edges. How much will you get paid total for mowing and trimming? Remember to show all your work.

8. You’re the manager of a county recreation center that has a 50 by 25 meter rectangular pool. Currently, there is an 1.5 meter cement walkway around the pool (see diagram.) The community is concerned about the safety of the walkway and would like to cover it with a non-slip rubber substance that costs $78 a square meter to be installed. The county has budgeted $15,000 for the project. Is that enough money to cover the walkway? Explain you answer.
**8.1f Self-Assessment: Section 8.1**

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept.

<table>
<thead>
<tr>
<th>Skill/Concept</th>
<th>Beginning Understanding</th>
<th>Developing Skill and Understanding</th>
<th>Deep Understanding, Skill Mastery</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Solve problems involving area and circumference of a circle with numeric measures.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Solve real-world and mathematical problems involving area and perimeter of two-dimensional objects composed of triangles and quadrilaterals.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Understand and explain the difference between perimeter and area.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Find area or perimeter of an object with algebraic measured lengths.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Solve problems involving scale or percent increase/decrease and area and/or perimeter.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Section 8.2: 2D Plane Sections from 3D Figures and 3D Measurement

Section Overview:

The goal of this section is to help students better understand a) attributes of various prisms, b) how those attributes effect finding the surface area and volume of different prisms, c) the relationship between measurements in 1, 2, and 3 dimensional figures and d) connect ideas of scale factor/percent change to ideas of surface area and volume. Students begin this section by examining three dimensional figures. They should observe which faces are parallel and/or perpendicular and which faces are the same size and shape. Students quickly move to taking cross-sections of various figures to notice what two dimensional shapes are generated by different types of cuts. Attention will be paid to when parallel plane sections generate surfaces that are the same and when they are different. The exercises in this section are designed to solidify students’ understanding of the algorithms for finding perimeter, area, and volume. Additionally, exercises should help students better understand units of measure for perimeter, area, and volume. Next, students examine the nets of 3D figures. Nets were introduced in 6th grade; in 7th grade, students extend their understanding by differentiating surface area from volume. Students will use their understanding of surface area and volume to solve various problems. Specifically, attention will be paid to scale factor and percent change in problems involving volume and surface area.

Concepts and Skills to be Mastered (from standards)

1. Describe the different ways to slice a 3D figure.
2. Describe the different 2D cross-sections that will result depending on how you slice the 3D figure.
3. Solve real-world and mathematical problems involving volume and surface area of three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Background knowledge for making and describing 2D cross-sections:

- Polyhedron prisms are three dimensional figures with at minimum two parallel polygonal faces of the same size and shape; remaining faces are rectangles of the same size and shape as each other but not necessarily as the bases (note that the base may be any polygon but the faces are always rectangles.)
- Cylinders have parallel circular bases of the same size (radii are equal) whose centers are aligned directly above each other (centers lie on a line that is perpendicular to both bases.)
- There are several ways to take cross-sections of a prism or cylinder (for the descriptions below, we assume the prism or cylinder is standing on one of its bases):
  - Parallel to the base
  - Perpendicular to the base
  - Slice at an angle (in a tilted direction) to the base
- “Equidistant” means an equal distance. “Vertices” are the corners of the figure.
8.2a Class Activity: 2D Plane Sections of Cubes and Prisms (play-dough & dental floss)

http://www.learner.org/courses/learningmath/geometry/session9/part_c/index.html  
http://www.shodor.org/interactivate/activities/CrossSectionFlyer/  

1. Mold a CUBE from play-dough.

<table>
<thead>
<tr>
<th>Perform the following cuts.</th>
<th>Sketch where you cut</th>
<th>Sketch the exposed surface(s).</th>
<th>NOTES</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Cut <em>parallel</em> to the base.</td>
<td><img src="https://via.placeholder.com/150" alt="Sketch" /></td>
<td><img src="https://via.placeholder.com/150" alt="Sketch" /></td>
<td></td>
</tr>
<tr>
<td>b. Cut parallel to the base again, but at a different distance from the base.</td>
<td><img src="https://via.placeholder.com/150" alt="Sketch" /></td>
<td><img src="https://via.placeholder.com/150" alt="Sketch" /></td>
<td>How does the new exposed surface compare to the previous surface?</td>
</tr>
</tbody>
</table>

What is true about all cuts of a cube parallel to the base?

What is true about other cuts of a cube NOT parallel to the base but through at least one lateral face?

c. Cut perpendicular to the base and parallel to a face.  
d. Cut perpendicular to the base and parallel to a face again, but at a different location on the cube.  

| ![Sketch](https://via.placeholder.com/150) | ![Sketch](https://via.placeholder.com/150) | |
| ![Sketch](https://via.placeholder.com/150) | ![Sketch](https://via.placeholder.com/150) | How does the new exposed surface compare to the previous surface? |

What is true about all cuts of a cube perpendicular to the base and parallel to a face?

What is true about cuts of a cube perpendicular to the base but not parallel to a face?
2. Mold a **CUBE** again. Perform a single cut to create the following:

<table>
<thead>
<tr>
<th>Create...</th>
<th>Sketch where you cut</th>
<th>NOTES</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Triangle</td>
<td><img src="image1.png" alt="Sketch" /></td>
<td></td>
</tr>
<tr>
<td>b. Square</td>
<td><img src="image2.png" alt="Sketch" /></td>
<td></td>
</tr>
<tr>
<td>c. Rectangle</td>
<td><img src="image3.png" alt="Sketch" /></td>
<td></td>
</tr>
<tr>
<td>d. Pentagon</td>
<td><img src="image4.png" alt="Sketch" /></td>
<td></td>
</tr>
<tr>
<td>e. Hexagon</td>
<td><img src="image5.png" alt="Sketch" /></td>
<td></td>
</tr>
<tr>
<td>f. Circle or Trapezoid</td>
<td><img src="image6.png" alt="Sketch" /></td>
<td></td>
</tr>
</tbody>
</table>
3. Mold a **RECTANGULAR PRISM**.

<table>
<thead>
<tr>
<th>Perform the following cuts.</th>
<th>Sketch where you cut</th>
<th>NOTES: describe the plane section and what might happen with other similar cuts.</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Cut parallel to the base.</td>
<td><img src="image1" alt="Sketch" /></td>
<td></td>
</tr>
<tr>
<td>b. Cut parallel to the base again, but at a different distance from the base.</td>
<td><img src="image2" alt="Sketch" /></td>
<td></td>
</tr>
</tbody>
</table>

What is true about all cuts of a rectangular prism parallel to the base?

What is true about cuts of a rectangular prism NOT parallel to the base?

c. Cut perpendicular to the base and parallel to a face. | ![Sketch](image3) |
d. Cut perpendicular to the base but not parallel to a face. | ![Sketch](image4) |

What is true about all plane sections of a rectangular prism perpendicular to the base and parallel to a face?

What is true about any plane section of a rectangular prism perpendicular to the base but not parallel to a face?
4. Mold a **SPHERE**.

<table>
<thead>
<tr>
<th>Perform the following cuts.</th>
<th>Sketch where you cut</th>
<th>NOTES: describe the plane section and what might happen with other similar cuts.</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Cut parallel to the table at different distances from the table.</td>
<td><img src="image1" alt="Sketch" /></td>
<td></td>
</tr>
<tr>
<td>b. Make cuts that are not parallel to the table.</td>
<td><img src="image2" alt="Sketch" /></td>
<td></td>
</tr>
</tbody>
</table>

What is true about all cuts of a sphere parallel to the table?

What is true about all cuts of a sphere NOT parallel to the table?

| a. Cut perpendicular to the table. | ![Sketch](image3) | |
| b. Cut perpendicular to the table again, but at a different location on the cube. | ![Sketch](image4) | |

What is true about all cuts of a sphere?
5. Make a cube and an rectangular prism like in exercises 1 and 3 again. Orient them as in the figure below:

What is the shape of the base of the cube and the rectangular prism?  
Will all cuts parallel to the base result in the same planar figure for the cube?  
Will all cuts parallel to the base result in the same planar figure for the rectangle prism?

Now rotate each 90° as shown in the figure below:

What is the shape of the base of the cube and the rectangular prism now?  
Will all cuts parallel to the base result in the same planar figure for the cube?  
Will all cuts parallel to the base result in the same planar figure for the rectangle prism?

6. Compare and contrast plane sections of a rectangular prism, cubes and spheres.
8.2a Homework: 3D Objects

Use your knowledge of each three-dimensional objects to answer the following questions.

Cubes
1. How many faces does a cube have?
2. What do you know about each face of a cube?
3. How many edges does a cube have?
4. How many vertices does a cube have?

Rectangular Prisms
5. How many faces does a rectangular prism have?
6. How many edges and vertices does a rectangular prism have?
7. How are a rectangular prism and a cube similar and different?

Sphere
8. Does a sphere have any edges or vertices?
9. What makes a sphere different from all the other 3D objects named above?
1. Mold a **Cylinder**, put the circular base on the table.

<table>
<thead>
<tr>
<th>Perform the following cuts.</th>
<th>Sketch where you cut</th>
<th>NOTES: describe the plane section and what might happen with other similar cuts.</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Cut parallel to the base.</td>
<td><img src="image" alt="Cylinder cut parallel to base" /></td>
<td></td>
</tr>
<tr>
<td>b. Make cuts not parallel to the base but through the lateral face of the cylinder.</td>
<td><img src="image" alt="Cylinder cut through lateral face" /></td>
<td></td>
</tr>
<tr>
<td>c. Cut perpendicular to the base.</td>
<td><img src="image" alt="Cylinder cut perpendicular to base" /></td>
<td></td>
</tr>
<tr>
<td>d. Make cuts that go through at least one base of the cylinder but are not perpendicular to the base.</td>
<td><img src="image" alt="Cylinder cut through base" /></td>
<td></td>
</tr>
</tbody>
</table>

What is true about all cuts of a cylinder parallel to the base?

What is true about all cuts of a cylinder NOT parallel to the base?

What is true about all cuts of a cylinder parallel to the base?

What is true about cuts of a cylinder perpendicular to the base?

What is true about any other cut (NOT parallel to the base or perpendicular to the base)?
2. Make a **Triangular Based Prism**. Put the triangle base on the table.

<table>
<thead>
<tr>
<th>Perform the following cuts.</th>
<th>Sketch where you cut</th>
<th>NOTES: describe the plane section and what might happen with other similar cuts.</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Cut parallel to the base.</td>
<td><img src="image1" alt="Sketch" /></td>
<td></td>
</tr>
<tr>
<td>b. Make cuts that are not parallel to the base but go through at least one face of the prism.</td>
<td><img src="image2" alt="Sketch" /></td>
<td></td>
</tr>
</tbody>
</table>

What is true about all cuts of a triangular prism parallel to the base?

| c. Cut perpendicular to the base. | ![Sketch](image3)     |                                                                                |

What is true about all cuts of a triangular prism parallel to the base?
3. Make a **Square-based Right Pyramid**. Put the square base on the table.

<table>
<thead>
<tr>
<th>Perform the following cuts.</th>
<th>Sketch where you cut</th>
<th>NOTES: describe the plane section and what might happen with other similar cuts.</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Make cuts parallel to the base.</td>
<td><img src="image" alt="Sketch" /></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>What do you notice about cuts of a square-based pyramid parallel to the base?</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. Cut perpendicular to the base and parallel to an edge between the base and a lateral face.</td>
<td><img src="image" alt="Sketch" /></td>
<td></td>
</tr>
<tr>
<td>c. Cut perpendicular to the base again, but at a different location on the pyramid.</td>
<td><img src="image" alt="Sketch" /></td>
<td></td>
</tr>
<tr>
<td>What do you notice about cuts of a triangular pyramid perpendicular to the table?</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4. Make a cylinder and a triangular prism like in exercises 1 and 2 again. Orient them as in the figure below:

What is the shape of the base of the cylinder?
What is the shape of the base of the triangular prism?
Will all cuts parallel to the base result in the same planar figure for the cylinder?
Will all cuts parallel to the base result in the same planar figure for the triangular prism?

Now rotate each 90° as shown in the figure below:

What is the shape of the base of the cylinder?
What is the shape of the base of the triangular prism?
Will all cuts parallel to the base result in the same planar figure for the cylinder?
Will all cuts parallel to the base result in the same planar figure for the triangular prism?
8.2b Homework: Area of Plane Sections

For each cross-section described below, state the shape of the cross-section and its area. Refer to previous class activities, as needed.

1. Imagine cutting a cube, parallel to the base.
   a. What shape is the plane section?
   b. If you cut a 5x5x5 inch cube parallel to any face, what will the area of the plane section be?

2. Imagine a right square based rectangular prism with edge lengths $\frac{3}{4} \times \frac{3}{4} \times 4 \frac{1}{2}$ inches.
   a. If you make a cut parallel to the square base, what will the cross section be?
   b. What will the area of the plane section described in “a” be?
   c. What will a the plane section be if the cut is made parallel to the face?
   d. What will the area of the plane section in “c” be?

3. Imagine cutting a sphere with diameter 10, parallel to the table.
   a. What shape will any plane section be?

4. Imagine cutting a cylinder of diameter 12.62cm and height 8cm, parallel to the base.
   a. What shape is the plane section and what is its area?
   b. What shape is the plane section if the cute is perpendicular to the base? What do you know about the figure?

5. Imagine a triangular prism:
   a. What shape is the plane section parallel to the base?
   b. What shape is a plane section perpendicular to the base?
   c. If the area of the plane section in “b” is 6.25 cm$^2$ and the height of the prism 5 cm. What was the length of the cut?

6. Imagine cutting a square based pyramid parallel to the base.
   a. What shape is the plane section?
   b. If the dimensions of the length and the width of the plane section are $\frac{3}{2}$ in. and $\frac{3}{2}$ in., what is the area of the plane section?
8.2c Class Activity: Nets of 3D Objects

*Review from 6th grade:* A net is a two dimensional figure that can be folded to make a three dimensional object. You can learn about a 3D object by examining its net.

Look at each net below. Determine if the net represents a right prism. If it does, state the name of the face you would choose as the base. If it does not, explain.

<table>
<thead>
<tr>
<th>NET</th>
<th>NET</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Net 1]</td>
<td>![Net 2]</td>
</tr>
<tr>
<td>1. Right Prism?</td>
<td>2. Right Prism?</td>
</tr>
<tr>
<td>![Net 3]</td>
<td>![Net 4]</td>
</tr>
<tr>
<td>![Net 5]</td>
<td>![Net 6]</td>
</tr>
</tbody>
</table>
10. The net to the right forms a cube.

   a. If each square face of the cube has a side length of 4.2 cm, what is the surface area of the cube?

   b. If each square face of the cube has a length of 4.2 cm, what is the volume of the cube?

   c. If you created a new cube with edge lengths that are 150% of the original cube (edge lengths of 4.2 cm), what would the new surface area and volume be?

   d. If you created a new cube with edge lengths that are 70% of the original cube (edge lengths of 4.2 cm), what would the new surface area and volume be?
8.2c Homework: Nets of 3D Objects

1. Which of the following nets make a cube?
   a. 
   b. 
   c. 
   d. 
   e. 
   f. 

2. Draw a net for the square based rectangular prism to the right:

3. Suppose the prism above has base edges of 8x8x12 inches. Find the surface area and volume of the prism. Show all your work.

4. Still using the prism above, find the new dimensions of the prism if you scaled it by 3/2. Then find the new surface area and volume.

5. Still using the prism above, find the new dimensions of the prism if you scale it by 2/3. Then find the new surface area and volume.
8.2c Additional Practice: Surface Area and Volume

1. Explain your procedure for finding surface area.

2. Explain your procedure for finding volume.

3. Find the surface area and volume of the right prism below. Note that the triangular bases are right triangles.

Find the surface areas and volume for each. You may wish to sketch the figure. Show work. Include units.

<table>
<thead>
<tr>
<th>Shape and Dimensions</th>
<th>Surface Area</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>4. Rectangular prism:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- length 10 in</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- width 8 in</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- height 5 in.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Rectangular prism:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- length 4 feet</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- width 2 feet</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- height 6.2 feet</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6. If the base area of a right rectangular prism is 28 cm² and the volume is 177.8 cm³, what is the height of the prism?

7. If the height of a square based prism is 13 in² and its volume is 637 in³, what is the length of each side of the base?

8. Give three possible edge lengths for a prism of volume 96 m³.
For each of the following problems, find the necessary measurements to answer all the questions.

1. A cube with side length 8 centimeters is enlarged so its side length is now 24 centimeters.
   a. By what scale factor did the side length increase? Show your work or justify your answer.

   b. By what scale factor will the surface area increase? Show your work or justify your answer.

   c. By what scale factor did the volume increase? Show your work or justify your answer.

2. A rectangular prism with side lengths 5 yd, 10 yd, and 15 yd is reduced by a scale factor resulting in new sides of 1 yd, 2 yd, and 3 yd.
   a. What was the scale factor for the reduction?

   b. What would be the new surface area and volume for the prism?

   c. By what scale factor did the surface and area each change?

3. A rectangular prism with side lengths 2 mm, 3 mm, and 5 mm is enlarged by some scale factor. The new volume of the prism is 240 mm$^3$.
   a. By what scale factor were the side lengths increase?

   b. What is the new length of each of the sides?

   c. What is the new surface area of the prism?
8.2d Homework: Growing and Shrinking Stuff

For each of the following problems, find the necessary measurements to answer all the questions.

1. A mini cereal box has the following dimensions: 4.5 in by 6 in by 2 in.
   a. If all the dimensions are doubled, will it require double the amount of cardboard to make the box? Why or why not?

   b. If all the dimensions are doubled, will it hold double the amount of cereal? Why or why not?

   c. If one of the dimensions is doubled, will it require double the amount of cardboard to make the box? Why or why not?

   d. If one of the dimensions is doubled, will it hold double the amount of cereal? Why or why not?

2. A container of chocolate milk mix has the following dimensions: a square base with sides of 6 in. and height 9 in.
   a. If all the dimensions are reduced by a scale factor of \( \frac{1}{3} \), will it require a third the amount of materials to make the container? Why or why not?

   b. If all the dimensions are reduced by a scale factor of \( \frac{1}{3} \), will it hold a third the amount of chocolate? Why or why not?

   c. If the length of each side of the base is reduced by a scale factor of \( \frac{1}{3} \), how much surface area and volume will it now have?

   d. What is the percent change of surface area and volume for “c”?
3. Refer back to the mini cereal box with dimensions, 4.5” by 6” by 2”.
   a. If all the dimensions are increased by 4 inches to: 8.5” by 10” by 6” how does this change the amount of cardboard needed to make the box?

   b. If all the dimensions are increased by a value of 4 inches, as described above, how will this change the amount of cereal the box can hold?
8.2e Project: Packing Packages

Have you ever notice that cereal generally comes in tall thin boxes and that laundry soap generally comes in short wide boxes? Why do you think they come as they do?

Think about your experiences with perimeter, surface area, and with volume in this chapter. What kind of box do you predict might hold the most and take the least amount of cardboard? Explain your thinking.

Suppose your favorite cereal comes in a box that is 24 cm. high, 20 cm. long, and 6 cm. wide. This box of cereal costs $4.35.

1. Draw a model of the box. Find the surface area and volume for the box. Show work and label answers.

The 24 x 20 x 6 cm dimensions of the box can also be thought of as a height of 24 cm and a girth (distance around the box) of 52 cm. If we add these two measures (height and girth), we get 76 cm.

For this project you will:

a. Draw a model of a box with a total girth plus height of no more than 76 cm that holds the most cereal possible with the least surface area possible.

b. Build the box as described in “a”.

c. Provide a table, graph or spreadsheet to show how you arrived at your dimensions remember (use only height and girth for your table, graph, or spreadsheet.)

d. Find the percent decrease in surface area for your new box from the original box.

e. Find the percent increase in volume for your new box from the original box.

f. Write two paragraphs about your project. In the first paragraph, state and justify how much you would charge for cereal in this box. In the second paragraph, explain why you think cereal does not come in the type of box you designed.
Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept.

<table>
<thead>
<tr>
<th>Skill/Concept</th>
<th>Beginning Understanding</th>
<th>Developing Skill and Understanding</th>
<th>Deep Understanding, Skill Mastery</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Describe the type of plane sections of right prisms and pyramids that result from different cuts.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Solve real-world and mathematical problems involving volume and surface area of three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Find the scale factor and/or percent change in surface area and/or volume when dimensions are changed.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Play Dough Recipe

Ingredients:
- 2 C flour
- 2 C warm water (food coloring is optional)
- 1 C salt
- 2 T oil
- 1 T cream of tartar

Directions:
In a medium pot, add water and oil together and stir. Then add the flour, salt, and cream of tartar. Keep stirring until all the ingredients are blended together and the mixture is not sticking to the sides of the pot. Then knead the mixture. For storage, keep play dough in plastic bags or tupperwear.