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Chapter 5: Geometry Part 1—Scale Drawings, Geometric Figures (3 weeks)

UTAH CORE Standard(s)

Draw construct, and describe geometrical figures and describe the relationships between them.
1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. 7.B.A.1
2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. 7.G.A.2
3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. 7.G.A.3

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.
4. Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. 7.G.B.4
5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. 7.G.B.5
6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. 7.G.B.6

VOCABULARY: angle, triangle inequality theorem, included angle, included side, congruent, equilateral triangle, isosceles triangle, scalene triangle, right triangle, acute triangle, obtuse triangle, corresponding parts, similar,

CONNECTIONS TO CONTENT:

Prior Knowledge: In elementary school students have found the area of rectangles and triangles. They have measured and classified angles, and drawn angles with a given measure. They have learned about circles informally, but haven’t learned rigorous definitions of Circumference and Area.

Future Knowledge: In 8th grade students will justify that the angles in a triangle add to 180° and will extend that knowledge to exterior angles and interior angles of other polygons. In 8th grade students will extend their understanding of circles to surface area and volumes of 3-D figures with circular faces. In 9th grade students will formalize the triangle congruence theorems (SSS, SAS, AAS, ASA) and use them to prove facts about other polygons. In 8th grade students will expand upon “same shape” (scaling) and extend that idea to dilation of right triangles and then to the slopes of lines. In 10th grade students will formalize dilation with a given scale factor from a given point as a non-rigid transformation (this will be when the term “similarity” will be define) and will solve problems with similar figures. The understanding of how the parts of triangles come together to form its shape will be deepened in 8th grade when they learn the Pythagorean Theorem, and in 11th grade when they learn the Law of Sines and Law of Cosines.
MATHEMATICAL PRACTICE STANDARDS (emphasized):

<table>
<thead>
<tr>
<th>Practice Standard</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Make sense of problems and persevere in solving them.</td>
<td>Students will analyze pairs of images to determine if they are exactly the same, entirely different or if they are the same shape but different sizes. With this information they will persevere in solving problems.</td>
</tr>
<tr>
<td>Reason abstractly and quantitatively.</td>
<td>Students will find scale factors between objects and use them to find missing sides. They will also note that proportionality exists between two sides of the same object. Students should move fluidly from $a:b = c:d \Rightarrow a:e = b:d$ etc. and understand why all these proportions are equivalent.</td>
</tr>
<tr>
<td>Construct viable arguments and critique the reasoning of others.</td>
<td>Students should be able to construct a viable argument for why two objects are scale versions of each other AND how to construct scale versions of a given object. Further students should be able to explain why, for example, if the scale between two objects is 5:3 why a length of 20 on the first object becomes 12 on the new object using pictures, words and abstract representations.</td>
</tr>
<tr>
<td>Model with Mathematics.</td>
<td>Students should be able to create a model (table of values, bar model, number line etc.) to justify finding a proportional values. Additionally, students should be able to start with a model for a proportional relationship and then write and solve a mathematical statement to find missing values.</td>
</tr>
<tr>
<td>Attend to Precision</td>
<td>Students should attend to units throughout. For example, if a scale drawing is 1mm = 3 miles, students should attend to units when converting from 4mm to 12 miles. Students should also carefully attend to parallel relationships, for example for two triangles with the smaller triangle having sides a, b, c and a larger triangle that is the same shape but different size with corresponding sides d, e, f, the proportion $a:d = b:e$ is equivalent to $a:b = d:e$ but sets up relationships in a different manner.</td>
</tr>
<tr>
<td>Look for and make use of structure</td>
<td>Students will link concepts of concrete representations of proportionality (bar models, graphs, table of values, etc.) to abstract representations. For example, if a length 20 is to be scaled down by a factor of 5:3 one can think of it as something times (5/3) is 20 OR 20 divided by 5 taken 3 times.</td>
</tr>
<tr>
<td>Use appropriate tools strategically.</td>
<td>By this point, students should set up proportions using numeric expressions and equations, though some may still prefer to use bar models. Calculators may be used as a tool to divide or multiply, but students should be encouraged to use mental math strategies where ever possible. Scaling with graph paper is also a good tool at this stage.</td>
</tr>
<tr>
<td>Look for and express regularity in repeated reasoning</td>
<td>Students should connect scale to repeated reasoning. For example if the scale is 1:3 than each length of the shorter object will be multiplied by 3 to find the length of the larger scaled object; then to reverse the process, one would divide by three.</td>
</tr>
</tbody>
</table>
5.0 Anchor Problem: What if you were a cartoon character?

Cartoon characters are supposed to be illustrated versions of human beings. In a way, we could think about a cartoon character as a scale drawing of a human.

What if Wreck-it Ralph were a scale drawing of you!? If you were Wreck-it Ralph but were your current height, how tall would your head be?

How big would your hands be?

How long would your legs be?
Section 5.1 Constructing triangles from given conditions

Section Overview: In this section, students discover the conditions that must be met to construct a triangle. Reflecting the core, the approach is inductive. By constructing triangles, students will note that the sum of the two shorter lengths of a triangle must always be greater than the longest side of the triangle and that the sum of the angles of a triangle is always 180 degrees. They then explore the conditions for creating a unique triangle: three side lengths, two sides lengths and the included angle, and two angles and a side length—whether or not the side is included. Although this inductive approach is not the “traditional” approach seen in recent education, it is the approach by which theorems were historically developed. In later grades, students will prove concepts they develop here.

In 7th grade, students are to learn about “scaling.” The concepts in this section are foundational to scaling and then lead to proportionality of objects that have the same “shape” in 5.2. In 8th grade, students will extend the idea of scaling to dilation and then in Secondary 1 to similarity. The word “similar” may naturally come up in discussions, however it will not be formally defined mathematically until Secondary 1. In this section emphasis should be made on conditions necessary to create triangles and that conditions are related to knowing sides and angles.

Concepts and Skills to be Mastered (from standards)

Geometry Standard 2:
1. Draw precise geometric figures based on given conditions
2. Discover the conditions necessary for a given set of angles or sides to make a triangle.
3. Explore conditions that determine unique triangles, multiple triangles, or no triangles.
5.1a Class Activity: Triangles and Labels—What’s Possible and Why?

Review Triangle: Write a definition and sketch at least one example for the following terms:

a. Acute triangle:  
b. Obtuse triangle:

c. Right triangle:  
d. Equilateral triangle:

e. Isosceles triangle:  
f. Scalene triangle:

Activity: The Engineer’s Triangle: **How many different triangles can an engineer make out of an 18 foot beam?**

You’re an engineer and you need to build triangles out of 18 foot beams. How many different triangles can you make with an 18 foot beam? What do you notice about the sides of different triangles? What do you notice about the angles of different triangles?

In groups of 2-4, cut out several “18 foot beams.” Your group’s task is to make different triangles with your “beams.” For each triangle you construct, use the table below to classify it by angles and sides.

Be careful to line up the exact corners of the strips, like this:  

![Exact corners example](image1)

NOT at the center, like this:

![Wrong corners example](image2)

32
GROUP RECORDING SHEET

For each triangle you construct: 1) record the length of each side, 2) the measure of each angle, 3) classify by angle and 4) classify by side. Pay attention to patterns. If you discover a pattern, write it down your conjecture.

<table>
<thead>
<tr>
<th>The lengths of each side.</th>
<th>The measure of each angle, to the nearest 5°.</th>
<th>Classify each triangle by side: Scalene, Isosceles, or Equilateral</th>
<th>Classify each triangle by angle: Right, Acute, or Obtuse</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B.</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>C.</td>
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<tr>
<td>D.</td>
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<td>E.</td>
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<tr>
<td>F.</td>
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<tr>
<td>G.</td>
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<tr>
<td>H.</td>
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<td></td>
</tr>
<tr>
<td>I.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>J.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>K.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Write the three numbers for the lengths of the side dimensions.</td>
<td>Write the three numbers for the angle measures, to the nearest 5°.</td>
<td>Write whether the triangle was Right, Acute, or Obtuse</td>
<td>Write whether the triangle was Scalene, Isosceles, or Equilateral</td>
</tr>
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<tr>
<td>L.</td>
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<tr>
<td>M.</td>
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<td>N.</td>
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<td>O.</td>
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<tr>
<td>P.</td>
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</tr>
<tr>
<td>Q.</td>
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<tr>
<td>R.</td>
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<tr>
<td>S.</td>
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<td></td>
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<tr>
<td>T.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Section 5.1 student materials

GROUP PAPER STRIPS

Instructions: Cut along the dotted lines to get strips 18 units long. Each unit represents one foot. For each trial, one member of your group will cut the strip into three pieces for the three side lengths of a possible triangle. Tape the triangle down when you’ve constructed it to help make the angle measuring easier.

Review: Using a protractor to find the measure of the angle.

Use the *inside* row, since this an obtuse angle. Thus, the angle is 100°, not 80°.
1. Is there more than one way to put together a triangle with three specific lengths?

2. What pattern do you see involving the sides of the triangle?

3. What pattern do you see involving the angles of the triangle?

4. For each group of three side lengths in inches, determine whether a triangle is possible. Write yes or no, and justify your answer.
   a. 14, 15 $\frac{1}{2}$, 2
   b. 1, 1, 1
   c. 7, 7, 16
   d. 4, 9, 5
   e. $6\frac{1}{3}$, 5, 4
   f. 3, 2, 1

5. If two side lengths of a triangle are 5 cm and 7 cm, what is the smallest possible integer length of the third side?

6. If two side lengths of a triangle are 5 cm and 7 cm, what is the largest possible integer length of the third side?
5.1a Homework: Triangle Practice

1. Explain the Triangle Inequality Theorem in your own words.

For #2-7, (a) Carefully copy the triangle using a ruler and protractor. Label the side lengths and angle measures for your new triangle. (b) Write an inequality that shows that the triangle inequality holds. (b) Classify the triangle as equilateral, isosceles, or scalene by examining the side lengths. (d) Classify the triangle as right, obtuse, or acute by examining the angle measures.

2.

3.

4.

5.
True or false. Explain your reasoning—if false, give a counter example.

8. An acute triangle has three sides that are all different lengths.

9. A scalene triangle can be an acute triangle as well.

10. An isosceles triangle can also be a right triangle.

11. If two angles in a triangle are 40° and 35°, the triangle must be acute.

12. An obtuse triangle can have multiple obtuse angles.

13. A scalene triangle has three angles less than 90 degrees.

14. A triangle with a 100° angle must be an obtuse triangle.
15. The angles of an equilateral triangle are also equal in measure.

16. Write whether the three given side lengths can form a triangle. If not, draw a sketch showing why it doesn’t work.
   a. 8, 1, 8  
   b. 3, 3, 3  
   c. 1, 4, 10  
   d. $1 \frac{1}{2}$, 5, $3 \frac{1}{2}$  
   e. 15, 8, 9  
   f. 2, 2, 15  
   g. 4, 3, 6.9  
   h. $\frac{1}{3}$, $\frac{1}{3}$, $\frac{1}{3}$

17. Anna used the scale on a map to calculate the distances “as the crow flies” (meaning the perfectly straight distance) from three points in Central America and the Caribbean islands, marked on the map to the right.
   a. According to Anna, how far is it from Jamaica to Panama if you don’t go through Honduras?
   b. According to Anna, how far is it from Jamaica to Panama if you do go through Honduras?
   c. Another way of stating the triangle inequality theorem is “the shortest distance between two points is a straight line.” Explain why Anna must have made a mistake in her calculations.
5.1b Class Activity: Building triangles given 3 measurements

Preparation for task: included angles.

1. Circle all the triangles with side lengths 8 and 5 and an included angle of 32°.

2. Circle all the triangles with side lengths 4 and 3, with a non-included angle of 50° adjacent to the side with length 3.

3. Circle all the triangles with two angles 90° and 28° with an included side of 6 units.
**Discussion questions:** What if you only have three pieces of information about a triangle, like two angle measurements and one side length? Is it possible that more than one triangle could be formed with that information?

Materials: Geogebra and 5.2 geogebra files.

Instructions: Open the file. Read the criteria that your triangle must satisfy. Drag the pieces to carefully form a triangle with the information required. If no triangle is possible, explain why. If only one triangle is possible, draw it and measure the remaining sides and angles using the tools in the program. If there is more than one way to make the triangle, draw both triangles.

<table>
<thead>
<tr>
<th>Trial #1: SSS</th>
<th>How many different triangles?</th>
<th>Draw the triangle(s), or write why no triangle is possible.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trial #2: SSS</td>
<td>How many different triangles?</td>
<td>Draw the triangle(s), or write why no triangle is possible.</td>
</tr>
<tr>
<td>Trial #3: SAS</td>
<td>How many different triangles?</td>
<td>Draw the triangle(s), or write why no triangle is possible.</td>
</tr>
<tr>
<td>Trial #4: ASA</td>
<td>How many different triangles?</td>
<td>Draw the triangle(s), or write why no triangle is possible.</td>
</tr>
<tr>
<td>Trial #5: SSA</td>
<td>How many different triangles?</td>
<td>Draw the triangle(s), or write why no triangle is possible.</td>
</tr>
<tr>
<td>Trial #6: AAS</td>
<td>How many different triangles?</td>
<td>Draw the triangle(s), or write why no triangle is possible.</td>
</tr>
</tbody>
</table>

Discussion notes:
5.1b Homework: Building triangles given three measurements

1. Use the grid below to draw and label \( \Delta ABC \) with the length of \( AB \) 5 units, the length of \( BC \) 7 units, and the measure of \( \angle ABC = 90^\circ \).

2. Is \( \angle ABC \) the included angle to the given sides? Explain.

3. What is the area of the triangle in #2? Justify your answer.

4. Will all your classmates who draw a triangle like #2 get a triangle with the same area? Explain.

5. Which triangle(s) has two sides 5 and 8 units, and a non-included angle of \( 20^\circ \) adjacent to the side of length 8?

6. Are the two triangles in #5 congruent? In other words, are they the exact same shape and size?
For #7-13, decide whether there are 0, 1, or more than one possible triangles with the given conditions. Use either a ruler and protractor, graph paper strips and protractor, or Geogebra to draw the triangles.

<table>
<thead>
<tr>
<th>7. A triangle with sides that measure 5, 12, and 13 cm.</th>
<th>8. A triangle with sides that two sides that measure 4 and 6, and an included angle of 120°.</th>
</tr>
</thead>
<tbody>
<tr>
<td>How many possible triangles?</td>
<td>How many possible triangles?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>9. A triangle with two angles 100° and 120°, and an included side of 2.</th>
<th>10. A triangle with two angles of 40° and 30°, and an included side of 6.</th>
</tr>
</thead>
<tbody>
<tr>
<td>How many triangles?</td>
<td>How many triangles?</td>
</tr>
</tbody>
</table>
11. A triangle with two sides of 5 and 7, and an included angle of 45°.

   How many triangles?

   If possible, what kind of triangle? Sketch, label.

12. A triangle with angles of 30° and 60°, and a non-included side of 5 units adjacent to the 60° angle.

   How many triangles?

   If possible, what kind of triangle? Sketch, label.

13. A triangle \( HIJ \) in which \( m\angle HIJ = 60°, \)
    \( m\angle JHI = 90°, \) and \( m\angle IHJ = 55°. \)

   How many possible triangles?

   If possible, what kind of triangle? Sketch, label.

14. A triangle \( ABC \) in which \( AB = 3 \) cm.,
    \( m\angle ABC = 60°, \) and \( m\angle BCA = 60°. \)

   How many possible triangles?

   If possible, what kind of triangle? Sketch, label.

15. Paul lives 2 miles from Rita. Rita lives 3 miles from the shopping mall. What are the shortest and longest distances Paul could live from the mall?

   (Whole number solutions)
   (Rational solutions)

   Draw and explain.
Activity: Cut out 5 different triangles, quadrilaterals, pentagons, and hexagons (all convex). Mark the angles and then tape them together as shown below. Use a protractor to find the measure of the sum of the angles and fill in the table.

<table>
<thead>
<tr>
<th>POLYGON</th>
<th>Number of Angles</th>
<th>Sum of Angles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle</td>
<td>3</td>
<td>180°</td>
</tr>
</tbody>
</table>
1) What pattern do you notice?

2) What do you expect the sum of the angles of an OCTAGON will be?

3) Cut out a convex octagon and test your conjecture.

4) Cut out a triangle and quadrilateral. Trace out the triangle on to a piece of grid paper. After you have traced it, rip the corners off and try to construct as many a new triangles as you can with the original angles. Do the same with the quadrilateral. What do you notice.
5.1c Homework: Sum of the Angles of a Polygon Exploration and 5.1 Review

1. Determine whether the three given side lengths can form a triangle. If not, explain why it doesn’t work.
   a) 5.1 inches, 3.24 inches, 8.25 inches
   b) 4 ¼ cm, 3 ¾ cm, 8 cm
   c) 6.01 cm, 5 ¼ cm, 11.22 cm
   d) 4 ½ m, 3 ¼ m, 8 cm

2. Your friend is having a hard time understanding how angle measures of 30°, 60°, 90° might create more than one triangle. Draw two different triangles that have those angle measures and explain why the two triangle are different.

3. Your friend is having a hard time understanding why knowing the lengths of two sides of a triangle and the measure of an angle not between the two sides may not be adequate information to construct a unique triangle. Draw an example where two sides and a non-included angle give two different triangles.

4. Predict the sum of the angles of a 10-agon.
5.1d Self-Assessment: Section 5.1

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept.

<table>
<thead>
<tr>
<th>Skill/Concept</th>
<th>Beginning Understanding</th>
<th>Developing Skill and Understanding</th>
<th>Deep Understanding, Skill Mastery</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Know the angle measure requirements of a triangle. (The sum of the angles of a triangle are always 180°)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Understand and explain the side length requirements of a triangle. (Explain why the sum of the two shorter sides of a triangle must be greater than the longest side.)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Understand and explain the angle and side requirements for constructing a unique triangle. (SSS, SAS, AAS, ASA and why AAA or SSA will not necessarily give a unique triangle.)</td>
<td></td>
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<tr>
<td>4. Given specific criteria, construct a triangle.</td>
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</tr>
</tbody>
</table>
Section 5.2: Scale drawings

Section Overview: The central idea of this section is scale and its relationship to ratio and proportion. Students will use ideas about ratio, proportion, and scale to a) change the size of an image and b) determine if two images are scaled versions of each other.

By the end of this section, students should understand that we can change the scale of an object to suit ourselves: we can make a map where 1 inch equals one mile, and we can draw a diagram of a house where 1 inch equals 10 feet, or a photo of an ant where 3 centimeters equals 1 mm. In each of these situations, the “shape characteristics” of the object remain the same, what has changed is size. Through explorations of scaling exercises, students will see that all lengths of the given object are changed by the same factor in the scaled representation; that factor is called the scale factor.

The term “similar” will not be defined in 7th grade; here students continue to develop an intuitive understanding of “the same shape,” so that the concept of similarity (introduced in 8th grade) is natural. Throughout this section students should clearly distinguish between two objects that are of the same shape and dimensions and objects that are scaled versions of each other. In particular students will come to understand that two polygonal figures that are scaled versions of each other have equal angles and corresponding sides in a ratio of \(a:b\) where \(a \neq b\). (In 8th grade, attention will be paid to figures that are of the same dimension and one can be superimposed on the other.) Students should also distinguish between saying the ratio of object A to object B is \(a:b\) and the scale factor from A to B is \(b/a\).

Students will learn to find the scale factor from one object to the other. Students should be able to fluidly go from a smaller object to a larger scaled version of the object or from a larger object to a smaller scaled version. In this section and through the chapter, abstract procedures will be emphasized for finding scale factors and/or missing lengths, however some students may prefer to continue to use bar models. Bar model strategies will become increasingly more cumbersome, so it will be important to help students transition to more efficient procedures. This can be done by helping students connect concrete models (i.e. bar models) to the algorithmic procedure as was done in Chapter 4.

Concepts and Skills to be Mastered (from standards)

Geometry Standard 1:
4. Use a scale or scale factor to find a measurement.
5. Find actual lengths and areas from a scale drawing, using a scale factor.
6. Create multiple scale drawings from the original model or drawing, using different scales.
5.2a Classwork: Scaling Triangles

Activity 1: Below is an image of a triangle. On the first grid provided, draw a triangle with sides lengths TWICE as long as this image (label the new vertices D, E, F) and on the other grid draw a triangle with side lengths HALF as long as this image (label the new vertices G, H, I.)

a) Describe the method you used to double the length of each side.

b) Describe the method you used to make each side have a length half the length of the original.

c) Do you think the angles of the new triangles are the same or different than the original triangle? Explain.
Activity 2: Exploring the relationship between triangles that have the same angle measures but side lengths that are different.

Which sides and angles correspond to each other? List corresponding angles and sides:

a) Describe how these triangles are similar and different:

b) What do you notice about the lengths of the sides of the two triangles?

Look back at Activity 1 in this section. Write the ratio of the original triangle to the triangle with side lengths that are twice the original and then write the ratio of the original triangle to the triangle with side lengths that are half as long as the original.
3. Fill in the following ratios. Write both the non-reduced and the reduced form of the ratio as a fraction.

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Non-reduced Form</th>
<th>Reduced Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) ( \frac{BC}{QR} )</td>
<td>( \frac{BC}{QR} )</td>
<td>( \frac{BC}{QR} )</td>
</tr>
<tr>
<td>b) ( \frac{AB}{QP} )</td>
<td>( \frac{AB}{QP} )</td>
<td>( \frac{AB}{QP} )</td>
</tr>
<tr>
<td>c) ( \frac{AC}{RP} )</td>
<td>( \frac{AC}{RP} )</td>
<td>( \frac{AC}{RP} )</td>
</tr>
<tr>
<td>d) Perimeter of ( \triangle ABC ) to Perimeter of ( \triangle PQR )</td>
<td>( \frac{\text{Perimeter of } \triangle ABC}{\text{Perimeter of } \triangle PQR} )</td>
<td>( \frac{\text{Perimeter of } \triangle ABC}{\text{Perimeter of } \triangle PQR} )</td>
</tr>
<tr>
<td>e) ( \frac{BC}{AB} )</td>
<td>( \frac{BC}{AB} )</td>
<td>( \frac{BC}{AB} )</td>
</tr>
<tr>
<td>f) ( \frac{QR}{QP} )</td>
<td>( \frac{QR}{QP} )</td>
<td>( \frac{QR}{QP} )</td>
</tr>
<tr>
<td>g) ( \frac{AC}{BC} )</td>
<td>( \frac{AC}{BC} )</td>
<td>( \frac{AC}{BC} )</td>
</tr>
<tr>
<td>h) ( \frac{RP}{QR} )</td>
<td>( \frac{RP}{QR} )</td>
<td>( \frac{RP}{QR} )</td>
</tr>
<tr>
<td>i) ( \frac{AC}{AB} )</td>
<td>( \frac{AC}{AB} )</td>
<td>( \frac{AC}{AB} )</td>
</tr>
<tr>
<td>j) ( \frac{RP}{QP} )</td>
<td>( \frac{RP}{QP} )</td>
<td>( \frac{RP}{QP} )</td>
</tr>
</tbody>
</table>

4. Some of the ratios above make comparisons of measurements in the same triangle. Other ratios make comparisons of measurements of corresponding parts of two different triangles. Put a star by the ratios that compare parts of two different triangles. What do you notice?

5. Since the corresponding parts have the same ratio, there is a scale factor from \( \triangle ABC \) to \( \triangle PQR \). The scale factor is the number you would multiply a length in \( \triangle ABC \) to get the corresponding length in \( \triangle PQR \). What is the scale factor from \( \triangle ABC \) to \( \triangle PQR \)? Explain how you arrived at your answer.

6. What is the scale factor from \( \triangle PQR \) to \( \triangle ABC \)? What is the mathematical relationship between the scale factors in #6 and #7?

7. The scale factors from \( \triangle ABC \) to \( \triangle PQR \) is an enlarging scale factor, and the other is a reducing scale factor. What would be the scale factor to keep a figure the same size?
8. Use a straight edge and protractor to construct $\triangle ABC$ then construct a new triangle, $\triangle DEF$, that is the same shape as $\triangle ABC$, but the scale factor from $\triangle ABC$ to $\triangle DEF$ is 2.

9. If the ratio of $\triangle HIJ$ to $\triangle DEF$ is $\frac{4}{3}$, (see $\triangle HIJ$ below), draw $\triangle DEF$. Show the length of each of the sides.

10. The ratio of $\triangle ABC$ to $\triangle DEF$ is 5:4. If $BC$ is 15, what is the length of $EF$? You may need to draw a diagram.
11. The two triangles below are scaled versions of each other. Find the measure of each of the angles of the two triangles below.

12. What is the ratio of $\triangle ABC$ to $\triangle XYZ$, as a fraction in lowest terms? What is the scale factor that takes $\triangle ABC$ to $\triangle XYZ$?

13. Which of the following proportions would be valid methods for finding the length of $BC$? Circle all that apply.
   a. $\frac{9.6}{6.4} = \frac{8.3}{BC}$
   b. $\frac{6.4}{BC} = \frac{8.3}{9.6}$
   c. $\frac{6.4}{BC} = \frac{9.6}{8.3}$
   d. $\frac{BC}{7.7} = \frac{8.3}{9.6}$

14. Write another different proportion that could be used to solve for the length of $BC$. Solve your proportion.

15. Write and solve a proportion to find the length of $XZ$. 

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5.2a Homework: Scaling Triangles

1. The measure of each of the sides of $\Delta DEF$ is given. Draw a $\Delta GHI$ that has side lengths that are three times as long as $\Delta DEF$

2. If the ratio of $\Delta BCD$ to $\Delta EFG$ is $3:5$ and the length of $\overline{BC}$ is 6”, what is the length of $\overline{EF}$? You may need to draw a diagram below.
For #3-4, a) solve for the variables by using proportions, given that each pair of triangles are scaled versions of each other, and b) state the scale factor between the two triangles, *assume we are looking for the scale factor that takes the triangle on the left to the one on the right*. Express all answers exactly, as mixed numbers when necessary. Figures are not necessarily drawn to scale. Show your work.

3.

4.
5.2b Class Activity: Solve Scale Drawing Problems, Create a Scale Drawing

1. Your sister wants a large poster version of a small drawing she made. She drew it on centimeter graph paper.

   a. What are the dimensions of her original picture?

   b. She wants the poster version to have a height of at least 2 feet. What scale should she use so that her poster is 2 feet tall?

   Distance in original : Distance in large poster

   c. The side of a square in the original picture is 1 cm long. How long will the side of a square be in the final poster?

   d. What will be the dimensions of the final poster?

2. Hal used the scale 1 inch = 6 feet for his scale model of the new school building. The actual dimensions of the building are 30 feet (height), by 120 feet (width) by 180 feet (length). What are the dimensions of his scale model?

3. On a separate sheet of grid paper, create the creature below so that it is a 1:3 enlargement of the original model. Write your strategy for calculating lengths to the right of the picture.
4. Ellie was drawing a map of her hometown using a scale of 1 centimeter to 8 meters.
   a. The actual distance between the post office and City Hall is 30 meters. What is the exact distance between those two places on Eddie's map?
   
   b. In her drawing, the distance from the post office to the library is 22 centimeters. What is the actual distance?

5. Allen made a scale drawing of his rectangular classroom. He used the scale \( \frac{1}{2} \) inch = 4 feet. His actual classroom has dimensions of 32 feet by 28 feet.
   a. What are the dimensions of his scale drawing of the classroom?
   
   b. The simplified unit ratio of classroom length : drawing length \( \left( \frac{\text{classroom length}}{\text{drawing length}} \right) \) is the scale factor for lengths. What is it?

   c. The simplified unit ratio of classroom area : drawing area \( \left( \frac{\text{classroom area}}{\text{drawing area}} \right) \) is the scale factor for areas. What is it?

   d. What is the mathematical relationship between the scale factor for lengths and the scale factor for areas?
6. Audrey wants to make a scale drawing of the stamp below. She makes a scale drawing where 1 cm in the drawing represents 3 cm on the stamp. Which of the following are true?
   a. The drawing will be \(\frac{1}{3}\) as wide as the stamp.
   
   b. The stamp will be 3 times as tall as the drawing.
   
   c. The area of the stamp will be 3 times as large as the drawing.
   
   d. The area of the stamp will be 6 times as large as the drawing.
   
   e. The area of the stamp will be 9 times as large as the drawing.

7. The following images are taken from four different maps or scale drawings, each shows a different way of representing scale:

<table>
<thead>
<tr>
<th>Scale ¼” = 1 Foot.</th>
<th>Scale 1:10</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Scale ¼” = 1 Foot." /></td>
<td><img src="image" alt="Scale 1:10" /></td>
</tr>
<tr>
<td>drawing size = 1 inch</td>
<td></td>
</tr>
<tr>
<td>real size = 1 meter</td>
<td></td>
</tr>
</tbody>
</table>

Which scale represents a drawing that has shrunk the most from the original? Justify your answer.

8. On the .25 inch grid on the next page, create a scale drawing of a classroom which is 15 meters by 10.5 meters. Determine a convenient scale for the grid below. In the scale drawing, include the following.
   c. A student desk which is .5 meters by .75 meters in real life.
   d. The entrance (1.25 meters wide) to the classroom (place where you choose).
   e. Two windows on one wall. The windows each 2.5 meters wide.
   f. Record the drawing scale below the drawing.
5.2b Homework: Class Activity: Solve Scale Drawing Problems, Create a Scale Drawing

1. On a map, Breanne measured the distance (as the crow flies) between Los Angeles, California and San Francisco, California at 2 inches. The scale on the map is \(\frac{1}{4}\) inch = 43 miles. What is the actual straight-line distance between Los Angeles and San Francisco?

2. Janie made a 2.5 inch scale model one of the tallest buildings in the world. Taipei 101. The scale for the model is \(\frac{1}{4}\) inch = 167 feet. Find the actual height of Taipei 101.

3. What scale was used to enlarge the drawing below? How do you know?
4. The scale of the drawing on the right is 1 unit = \( \frac{1}{4} \text{ foot} \). Use the grid below to draw a new scale drawing where 1 unit = \( \frac{1}{2} \text{ foot} \). 

5. On the .25 inch grid on the next page, create a scale drawing of a living room which is 9 meters by 6.25 meters. In the scale drawing include the following:
   a. Two windows on one of the walls. Each window is 1.25 meters wide.
   b. An entrance (1.5 meters wide) from the side opposite the windows.
   c. A sofa which is 2 meters long and .75 meters wide.
   g. Record the drawing scale below the grid.
5.2c Class Activity: Scale Factors and Area

The scale drawing of a house is shown. The scale is 1 unit : 2 feet. Any wall lines that are between units are exactly halfway.

1. The balcony off the bedroom has dimensions in the drawing of 7 x 2. Complete the table, including the appropriate units:

<table>
<thead>
<tr>
<th>Drawing</th>
<th>Balcony length</th>
<th>Balcony width</th>
<th>Balcony area</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 units</td>
<td>2 units</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. What is the scale factor to get from units in the drawing to feet in the house?

3. What is the scale factor to get from square units in the drawing to square feet in the house?

4. If the architect includes a bench on the balcony that has dimensions 2.5 x 1 units, what are the dimensions of the bench in the house?
5. Complete the table for the bathroom, including the appropriate units:

<table>
<thead>
<tr>
<th></th>
<th>Bathroom length</th>
<th>Bathroom width</th>
<th>Bathroom area</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Drawing</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Actual house</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6. What is the ratio of \( \frac{\text{area in house}}{\text{area in drawing}} \)?

7. If the backyard in the drawing has an area of 150 square units, how big is the area of the actual backyard of the house?

8. If the walkway to the entryway is 6.5 units long in the drawing, how long is the walkway on the house?

9. Use the drawing to determine how wide each interior door in the house is.

10. Approximately how long is each bed in the house?

11. By counting the number of square units in the master bedroom in the drawing, calculate the area of the master bedroom in the house. Include the furniture in the area.

12. Challenge: What is the total square footage of the house, including the balconies? Show your work, labeling the expressions for each step with what they represent in the house.
5.2c Homework: Scale Factors and Area

1. Mouse’s house is very small. His living room measures 2 feet by 4 feet. Draw his living room on the grid. Label the dimensions and the area.

2. Double-Dog’s living room dimensions are double Mouse’s living room. Draw his living room on the grid. Label the dimensions and the area.

3. Triple-Threat-Tiger’s house is triple the dimensions of Double-Dog’s. Draw Tiger’s living room on the grid. Label the dimensions and the area.

4. Fill in the table below with the dimensions and areas of the living rooms from the sketches above.

<table>
<thead>
<tr>
<th>Homeowner</th>
<th>Living Room Dimensions (Length and Width)</th>
<th>Living Room Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mouse</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Double-Dog</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Triple-Threat-Tiger</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. Compare measurements for Mouse and Double-Dog’s living rooms. Use scale factor in the comparison.
   a. the dimensions
   b. the area

   a. the dimensions
   b. the area

7. Compare Mouse and Triple-Threat-Tiger’s living rooms. Use scale factor in the comparison.
   a. The dimensions
   b. the area
Generalize a rule related to the scale factors of dimensions and area. You might use “if…, then….”

For example: “If the dimensions of an object are multiplied by ______, the area will be multiplied by ______.”

8. If the area of Mouse’s kitchen is 12 square feet, and the dimensions of Double-Dog’s kitchen are twice as big as the dimensions of Mouse’s, what will the area of Double-Dog’s kitchen be?

9. If the dimensions of Triple-Threat-Tiger’s kitchen are three times the dimensions of Double-Dog’s, what will the area of Triple-Threat-Tiger’s kitchen be?

10. If the area of Mouse’s bathroom is 5 square feet, the dimensions of Double-Dog’s bathroom are twice Mouse’s, and Triple-Tiger’s are three times Double-Dog’s, what will the area of Triple-Threat-Tiger’s bathroom be? Is there a shortcut?

11. Ms. Herrera decided to shrink a picture so that it would fit on a page with some text. She went to the copy machine and pushed the 50% button, meaning that the dimensions of the paper would be half as big as normal.
   a. If the original dimensions of the picture were 8.5 in. X 11 in., what will be the dimensions of the new picture?

   b. Draw the original and new picture on the grid. Label the dimensions.

   c. If you know the area of the original picture, how might you figure out the area of the smaller picture (besides multiplying length and width)?

12. Let’s say that Mouse, Double-Dog and Triple-Threat-Tiger all have swimming pools. What would you predict about the scale factor of the volume as related to double or triple dimensions? Prove or adjust your prediction—say that Mouse has a pool which is 1 foot deep and 2 feet by 3 feet.
5.2d Class Activity: Constructing Scale Drawings

Task: Making a scale drawing of the top of your desk

- Use measuring tools and scissors to cut a piece of letter sized paper (you could use graph paper if you want) to be a scale model of the top of your desk.
- Write the scale on the corner of the paper.
- Place at least 3 items on your desk. For example, a pencil, eraser, book, soda can, cell phone, etc.
- Fill in the table of values below with measurements or calculations. You can choose what to measure for the five blank rows. The blanks in the column headings are to write what units you measured in (cm, inches, mm, units of graph paper, etc.)

<table>
<thead>
<tr>
<th>Item measured</th>
<th>Real measurement in _________</th>
<th>Measurement in scale drawing in _________</th>
</tr>
</thead>
<tbody>
<tr>
<td>A) Short side of top of desk</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B) Long side of top of desk</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Use measuring tools to draw your three items, to scale and in the same position as they are on your desk, on your scale drawing.
- Graph the information from your table on the grid on the next page, labeling each point with the letter A-G from the row of the table. Be sure to label the axes.
5.2d Homework: Constructing Scale Drawings

Task: Making a scale drawing as a class.

Below is an image of an ant (NylanderiaPubens Worker ant.)
- Use measuring tools to make a scale drawing of the ant picture below.
- Determine your scale.
5.2e Extra Task: **Planning a Playground** (Illuminations)(area/perimeter, scale model)

http://illuminations.nctm.org/LessonDetail.aspx?id=L763
Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept.

<table>
<thead>
<tr>
<th>Skill/Concept</th>
<th>Beginning Understanding</th>
<th>Developing Skill and Understanding</th>
<th>Deep Understanding, Skill Mastery</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Draw a scaled version of either a triangle, other polynomial, or other object given lengths of sides</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Find a measure of a scaled object given the scale factor and measure from the original</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Find the scale factor between two objects that are the same shape but different sizes.</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>4. Use proportional reasoning in explaining and finding missing sides of objects that are the same shape but different sizes.</td>
<td></td>
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</tr>
</tbody>
</table>
Section 5.3: Solving problems with circles

Section Overview: In this section circumference and area of a circle will be explored from the perspective of scaling. Students will start by measuring the diameter and circumference of various circles and noting that the ratio of the circumference to the radius is constant ($2\pi$). This should lead to discussions about all circles being scaled versions of each other. Next students will “develop” an algorithm for finding the area of a circle using strategies used throughout mathematical history. In these explorations, students should discuss two ideas: 1) cutting up a figure and rearranging the pieces so as to preserve area, and 2) creating a rectangle is a convenient way to find area. Students will connect the formula for finding the area of a circle ($\pi r^2$) to finding the area of a rectangle/parallelogram where the base is $\frac{1}{2}$ the circumference of the circle and the height is the radius ($A=Cr/2$).

Students end the section by applying what they have learned to problem situations. In Chapter 6 students will use ideas of how circumference and area are connected to write equations to solve problems but in this section students should solve problems using informal strategies to solidify their understanding.

Concepts and Skills to be Mastered (from standards)

Geometry Standard 4: Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
5.3a Classwork: How many diameters does it take to wrap around a circle?

1. Create at least 5 different circles using either a) manual construction: a compass, various circular objects you can trace, or string, OR b) technology: geogebra, etc. (technology will allow for far more accurate measurement.) Then, measure the circumference and diameter of the circle; fill in the table below.

<table>
<thead>
<tr>
<th>Measure of the diameter in _______ units</th>
<th>Measure of the circumference in _______ units (must be the same units as the diameter)</th>
<th>Ratio of circumference : diameter (C/d), as a decimal rounded to the nearest hundredth. Note: C represents circumference and d represents diameter.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tr>
</tbody>
</table>

2. What do you notice about the values in the third column?

3. If you made a huge circle the size of a city and measured the diameter and circumference, would the ratio of circumference to diameter (C/d) be consistent with the other ratios in the third column of the table? Justify your answer using what you learned from the previous section.

4. If you know the diameter of a circle is 5 inches, what is the approximate measure of the circumference? Justify your answer.
5. Write and justify a formula for circumference, in terms of the diameter.

6. Write and justify a formula for the circumference, in terms of the radius.

7. For each of the three circles below, calculate the circumference of the circle. Express your answer both in terms of \( \pi \), and also as an approximation to the nearest tenth.

8. If the circumference of a circle is \( 8\pi \) (approximately 25.1) inches, which of the following is true? Rewrite false statements to make them true.
   a. The ratio of Circumference: Diameter is 8.
   b. The radius of the circle is \( \frac{1}{2} \) the circumference.
   c. The diameter of the circle is twice the radius.
   d. The radius of the circle is 8 inches.
   e. The diameter of the circle is 8 inches.
9. The circumference of 5 objects is given. Calculate the diameter of each object, to the nearest tenth of a unit.

- Circumference of bike wheel: 76.9"
- Circumference of car tire: 78.1"
- Circumference of lid: 11"
- Circumference of top of garbage can: 50"
- Circumference of plate: 31"

10. When a unicyclist pedals once, the wheel makes one full revolution, and the unicycle moves forward the same distance along the ground as the distance around the edge of the wheel. If Daniel is riding a unicycle with a diameter of 20 inches, how many times will he have to pedal to cover a distance of 50 feet? Show all your work.
5.3a Homework: How many diameters does it take to wrap around a circle?

1. Identify 5 circular objects around the house (can foods, door knobs, cups, etc.) Find the measure of the objects’ diameter and then calculate its circumference. Put your results in the table below:

<table>
<thead>
<tr>
<th>Description of item</th>
<th>Diameter (measured)</th>
<th>Circumference (calculated)</th>
<th>Ratio of C: d (calculated) to the nearest hundredth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
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<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

2. What is the exact ratio of the circumference to the diameter of every circle?

3. If the radius of a circle is 18 miles,
   a. What is the measure of the diameter?
   b. What is the measure of the circumference, exactly in terms of pi?
   c. What is the approximate measure of the circumference, to the nearest tenth of a mile?

4. For each of the three circles below, calculate the circumference. Express your answer both in terms of pi, and also as an approximation to the nearest tenth.

radius = 1.5 cm
circumference = C
Solve for C.

Diameter = 5 cm
circumference = C
Solve for C.

radius = 9 units
circumference = C
Solve for C.
The decimal for \( \pi \) starts with 3.141592653589… Which fraction is closest to \( \pi \)? (Note: there is no fraction that is exactly equal to \( \pi \).)

\[ \begin{array}{c|c|c|c|c|c} a) \frac{3}{4} & b) \frac{1}{5} & c) \frac{1}{7} & d) \frac{1}{6} & e) \frac{1}{8} \end{array} \]

5. If the circumference of a circle is \( 20\pi \) feet, which of the following statements are true? Rewrite false statements to make them true.
   a. The circumference of the circle is exactly 62.8 feet.
   b. The diameter of the circle is 20 feet.
   c. The radius of the circle is 20 feet.
   d. The ratio of Circumference: diameter of the circle is \( \pi \).
   e. The radius of the circle is twice the diameter.

6. The circumference of 5 objects is given. Calculate the diameter of each object, to the nearest tenth of a unit.

7. Three tennis balls are stacked and then tightly packed into a cylindrical can. Which is greater: the height of the can, or the circumference of the top of the can? Justify your answer.
5.3b Classwork: Area of a Circle

Activity: Circle Area

History: Methods for computing the area of simple polygons was known to ancient civilizations like the Egyptians, Babylonians and Hindus from very early times in Mathematics. But computing the area of circular regions posed a challenge. Archimedes (287 BC – 212 BC) wrote about using a method of approximating the area of a circle with polygons. Below, you will try some of the methods he explored for finding the area of a circle of diameter 6 units.

a) Estimate the area of the circle by counting the number of square units in the circle.

b) Estimate the area of the circle by averaging the inscribed and circumscribed squares.

<table>
<thead>
<tr>
<th>a) Estimated area = __________________</th>
<th>b) Estimated area = __________________</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Diagram" /></td>
<td><img src="image2.png" alt="Diagram" /></td>
</tr>
</tbody>
</table>

How many radius squares cover the same area as the circle? __________
Johannes Kepler (1571-1630) tried a different approach: he suggested dividing the circle into “isosceles triangles” and then restructuring them into a parallelogram. Refer to the text for more information about this approach.

Cut the circle into eighths. Then fit and paste the eighths into a long line (turn the pie pieces opposite ways) to create a “parallelogram.”
The figure to the left shows the same circle of radius 3 as the previous example, but this time cut into 10 wedges. How will this parallelogram compare to the one created with 8 wedges above?

Will the area created by reorganizing the pieces be the same or different than the original circle? Explain.

In the next diagram, the same circle of radius 3, but this time it’s cut into 50 wedges. Again it is packed together into a parallelogram.

Highlight the circumference of the circle. Then highlight where the circumference is found in the new diagram. Explain why the base of the “parallelogram” is half the circumference of the circle.

Highlight the circumference of the circle. Then highlight where the circumference is found in the new diagram. Explain why the base of the “parallelogram” is half the circumference of the circle.

Use the figure and what you know about the area of a rectangle to write an expression for the area of the circle.
1. Estimate the area of the circle in square units by counting.

2. Use the formula for the area of a circle to calculate the exact area of the circle above, in terms of $\pi$.

3. Calculate the area for #2 to the nearest square unit. How accurate was your estimate in #1?

4. Calculate the area of each circle. Express your answer both exactly (in terms of $\pi$) and approximately, to the nearest tenth of a unit.
5. A certain earthquake was felt by everyone within 50 kilometers of the epicenter in every direction.
   a. Draw a diagram of the situation.
   
   b. What is the area that felt the earthquake?

6. There is one circle that has the same numeric value for its circumference and its area (though the units are different.) Use any strategy to find it. Hint: the radius is a whole number.

7. Explain the difference in the units for circumference and area the circle in #6

8. Draw a diagram to solve: A circle with radius 3 centimeters is enlarged so its radius is now 6 centimeters.
   a. By what scale factor did the circumference increase? Show your work or justify your answer.
   
   b. By what scale factor did the area increase? Show your work or justify your answer.
   
   c. Explain why this makes sense, using what you know about scale factor.

9. How many circles of radius 3” can you fit in a circle with radius 12” (if you could cut up the smaller circles to tightly pack them into the larger circle with no gaps)? See the image below. Justify your answer.
9. Calculate the radius for each circle whose area is given in the table (the first entry is done for you). Then graph the values on a coordinate plane, with the radius on the \( x \) axis and the approximate area on the \( y \) axis.

<table>
<thead>
<tr>
<th>Radius of circle</th>
<th>6 units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area of circle</td>
<td></td>
</tr>
<tr>
<td>( 36\pi \text{ un}^2 )</td>
<td>( \approx 113 \text{ un}^2 )</td>
</tr>
<tr>
<td>( 16\pi )</td>
<td>( \approx 50 \text{ un}^2 )</td>
</tr>
<tr>
<td>( 81\pi \text{ un}^2 )</td>
<td>( \approx 254 \text{ un}^2 )</td>
</tr>
<tr>
<td>( 25\pi \text{ un}^2 )</td>
<td>( \approx 79 \text{ un}^2 )</td>
</tr>
<tr>
<td>( 9\pi \text{ un}^2 )</td>
<td>( \approx 28 \text{ un}^2 )</td>
</tr>
<tr>
<td>( 64\pi \text{ un}^2 )</td>
<td>( \approx 201 \text{ un}^2 )</td>
</tr>
<tr>
<td>( 49\pi \text{ un}^2 )</td>
<td>( \approx 154 \text{ un}^2 )</td>
</tr>
<tr>
<td>( 4\pi \text{ un}^2 )</td>
<td>( \approx 12 \text{ un}^2 )</td>
</tr>
</tbody>
</table>

10. Is the radius of a circle proportional to the area of the circle? Justify your answer.
5.3b Homework: Area of a Circle

1. Estimate the area of the circle in square units by counting.

2. Use the formula for the area of a circle to calculate the exact area of the circle above, in terms of pi.

3. Calculate an approximation for the area expression from #2, to the nearest square unit. How accurate was your estimate in #1?

4. Calculate the area of each circle. Express your answer both exactly (in terms of pi) and approximately, to the nearest tenth of a unit.
5. The strongest winds in Hurricane Katrina extended 30 miles in all directions from the center of the hurricane.
   a. Draw a diagram of the situation.
   b. What is the area that felt the strongest winds?

6. By calculating the areas of the square and the circle in the diagram, determine how many times larger in area the circle is than the square.

7. Draw a diagram to solve: A circle with radius 8 centimeters is enlarged so its radius is now 24 centimeters.
   a. By what scale factor did the circumference increase? Show your work or justify your answer.
   b. By what scale factor did the area increase? Show your work or justify your answer.
   c. Explain why this makes sense, using what you know about scale factor.
8. How many circles of radius 1” could fit in a circle with radius 5” (if you could tightly pack in the area with no gaps)? Justify your answer.

9. Calculate the radius for each circle whose circumference is given in the table (the first entry is done for you). Then graph the values on a coordinate plane, with the radius on the x axis and the approximate circumference on the y axis.

<table>
<thead>
<tr>
<th>Radius of circle</th>
<th>Circumference of circle</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 units</td>
<td>8\pi \text{ un}</td>
</tr>
<tr>
<td></td>
<td>\approx 25 \text{ un}</td>
</tr>
<tr>
<td></td>
<td>10\pi \approx 31 \text{ un}</td>
</tr>
<tr>
<td></td>
<td>2\pi \approx 6 \text{ un}</td>
</tr>
<tr>
<td></td>
<td>16\pi \approx 50 \text{ un}</td>
</tr>
<tr>
<td></td>
<td>6\pi \approx 19 \text{ un}</td>
</tr>
<tr>
<td></td>
<td>18\pi \approx 57 \text{ un}</td>
</tr>
<tr>
<td></td>
<td>4\pi \approx 13 \text{ un}</td>
</tr>
<tr>
<td></td>
<td>12\pi \approx 38 \text{ un}</td>
</tr>
</tbody>
</table>

10. Is the radius of a circle proportional to the circumference of the circle? Justify your answer.
Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept.

<table>
<thead>
<tr>
<th>Skill/Concept</th>
<th>Beginning Understanding</th>
<th>Developing Skill and Understanding</th>
<th>Deep Understanding, Skill Mastery</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Explain the relationship between diameter of a circle and either its circumference or area.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Explain the algorithm for finding circumference or area of a circle.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Find the circumference or area of any circle given diameter or radius; or given circumference or area determine diameter or radius.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Section 5.4: Angle relationships

Section Overview: In this section students will learn and begin to apply angle relationships for vertical angles, complementary angles and supplementary angles. They will practice the skills learned in this section further in Chapter 6 when they write equations involving angles. Students will also use concepts involving angles to relate scaling of triangles and circles. At the end of this section there is a review activity to help students tie concepts together.

Concepts and Skills to be Mastered (from standards)

Geometry Standard 5: Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
5.4a Classwork: Special angle relationships

This diagram is a regular pentagon with all its diagonals drawn and all points labeled.

1. How many non-overlapping angles are in the diagram?

2. There are groups of angles that all have the same measure. For example, \( \angle DEG \) and \( \angle AEF \) have the same measure. How many different measures of angles are there in the diagram? Use a protractor.

3. Name at least five vertical angle pairs. Use a different colored pencil to mark each pair in the diagram above.

4. What seems to be the relationship between measures of two vertical angles? Draw another pair of vertical angles below by constructing two intersecting lines and measure the two angles in the vertical pair. Does this example support your conjecture?

Vertical Angles: To lines that intersect form vertical angles. Vertical angles are pairs of angles that are always opposite one another (rather than adjacent to each other). For example, \( \angle EFG \) and \( \angle AFJ \) are vertical angles.

Adjacent Angles: Two angles are adjacent if they have a common ray (side) and vertex. For example, \( \angle EFG \) and \( \angle EFA \) are adjacent angles.
5. Find and name at least 5 pairs of supplementary angles in the diagram. Use a different color to mark each pair in the diagram.

6. For each pair of intersecting lines below, find the three missing measures of angles formed. Justify your answer in the table.

<table>
<thead>
<tr>
<th>Angle</th>
<th>Measure of angle</th>
<th>Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \angle CEB )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \angle DEB )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \angle AEC )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Angle</th>
<th>Measure of angle</th>
<th>Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \angle CEB )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \angle DEB )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \angle AED )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
7. For this rectangle with diagonals drawn in, there is one place where you can see supplementary and vertical angles. Use a protractor to measure one of the angles, and then calculate the measures of the other three angles that have vertex at E using facts about vertical and supplementary pairs.

<table>
<thead>
<tr>
<th>Angle</th>
<th>Measure of angle</th>
<th>Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>∠CEB</td>
<td></td>
<td></td>
</tr>
<tr>
<td>∠DEB</td>
<td></td>
<td></td>
</tr>
<tr>
<td>∠AED</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

8. Find and name at least five more pairs of complementary angles in the figure above. Use a different highlighter to mark each pair on the diagram.

∠EAB and ∠DAE are complementary because their measures add to 90°. When complementary angles are adjacent, you can see the right angle that is formed by the outside rays. However, complementary angles don’t need to be adjacent; as long as their measures add to 90 degree, two angles form a complementary pair.
Review: What is the sum of all the angles in a triangle?

9. Consider a right triangle. What seems to be true about the two non-right angles in the triangle? Use the examples below, or draw your own examples to help with your conjecture.

10. Look around the room you’re in right now. Find examples of angles in the furniture, tiles, posters, etc. Can you see any complementary angles? Can you see supplementary angles? Can you see vertical angles? Draw sketches for at least three angle pairs you find.
For #11-12, use properties of complementary, supplementary, and vertical angles to find missing measures.

11. Two pair of seesaws sit unused at a playground, as shown. \( \angle EGF \) has a measure of 140°.
   
   a. Which angle is vertical to \( \angle EGF \)?
   
   b. What is the measure of \( \angle CGH \)?
   
   c. Name an angle that is supplementary to \( \angle EGF \).
   
   d. What is the measure of \( \angle CGE \)?
   
   e. What is the measure of \( \angle HGF \)?

12. In the diagram below, \( \angle ADB \) is a right angle. The figure is formed by 3 intersecting lines.

   Fill in the measures and justifications in the table:

<table>
<thead>
<tr>
<th>Angle</th>
<th>Measure</th>
<th>Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \angle CDA )</td>
<td>90°</td>
<td>Supplementary to ( \angle ADB ), which is 90.</td>
</tr>
<tr>
<td>( \angle ADG )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \angle GDB )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \angle BDF )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \angle EDC )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5.4a Homework: Special angle relationships

1. Find at least one example of each angle relationship in the diagram. Name the angle pairs below, and highlight the pairs of angles in the diagram, use a different color for each relationship.

a) Vertical angles

b) Supplementary angles

c) Complementary angles
2. For each figure of two intersecting lines, calculate the three missing measures, justifying your answer.

<table>
<thead>
<tr>
<th>Angle</th>
<th>Measure of angle</th>
<th>Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>∠CEB</td>
<td></td>
<td></td>
</tr>
<tr>
<td>∠DEA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>∠BED</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Angle</th>
<th>Measure of angle</th>
<th>Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>∠LOM</td>
<td></td>
<td></td>
</tr>
<tr>
<td>∠MOK</td>
<td></td>
<td></td>
</tr>
<tr>
<td>∠NOL</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Angle</th>
<th>Measure of angle</th>
<th>Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>∠PST</td>
<td></td>
<td></td>
</tr>
<tr>
<td>∠RSQ</td>
<td></td>
<td></td>
</tr>
<tr>
<td>∠QST</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3. For the figure formed by three intersecting lines, calculate the four missing measures, justifying your answer.

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
\text{Angle} & \angle AEF & \angle AEC & \angle AED & \angle DEG & \angle CEB & \angle FEG \\
\hline
\text{Measure} & 52^\circ & & & & & \\
\hline
\text{Justification} & \text{Vertical to } \angle GEB & & & & & \\
\hline
\end{array}
\]

4. Refer to the figures below.

\[m\angle AZU = \text{because}\]

\[m\angle KJH = \text{because}\]
5. Fill in the missing angle measurements in the table, and give a justification for each measurement.

<table>
<thead>
<tr>
<th>Angle</th>
<th>$\angle ADG$</th>
<th>$\angle GDB$</th>
<th>$\angle BDH$</th>
<th>$\angle CDH$</th>
<th>$\angle CDE$</th>
<th>$\angle CDA$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measure</td>
<td>57°</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Justification</td>
<td>Vertical to $\angle EDH$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For #5-7, draw a diagram to illustrate the situation, and then choose the correct answer.

5. If $\angle G$ is complementary to $\angle H$, and $m \angle H = 20^\circ$, then $\angle G$ must be:
   - a. Obtuse
   - b. Acute
   - c. Right

6. If $\angle B$ is supplementary to $\angle C$, and $m \angle C = 90^\circ$, then $\angle B$ must be:
   - d. Obtuse
   - e. Acute
   - f. Right

7. If $\angle D$ is vertical to $\angle E$, and $m \angle E = 115^\circ$, then $\angle E$ must be:
   - g. Obtuse
   - h. Acute
   - i. Right
5.4b Classwork: Circles, Angles, and Scaling

Examine the figures below. $\overline{BC}$ has a length of 5 units.

1. Write a complete sentence explaining why the figures are scale drawings of each other. You may use a protractor. Use words like corresponding angles, scale factor, corresponding sides, circle.

2. Remember that you have been given that $\overline{BC}$ has length 5. What is the radius of each circle? Justify your answer.

3. Calculate the area of each triangle.

4. Classify the triangles by their sides and angles.

5. Name two pairs of complementary angles in the figures.
6. What is the area of each circle?

7. How many of the smaller circle would fit inside the bigger circle, if you could put all the area in without overlapping and with no empty space?

8. What is the circumference of each circle?

9. How many of the circumferences of the smaller circle equal the bigger circumference?

10. Fill in the blanks: When you enlarge a figure with a scale factor of two, the side lengths and circumference____________, the areas ________________, and the angles _________________.

11. You could construct other figures that are similar to these two figures with a different scale factor. What would be the dimensions of the triangle with scale factor $\frac{1}{2}$ from the figure on the left? What would be the radius of the circle?

12. Follow the steps below to create another figure in the space below:
   a. Make a dot for the center of a circle, and use a compass to construct a circle around that dot.
   b. Use a straightedge to draw in a diameter with endpoints labeled A and B.
   c. Choose any point on the edge of the circle, and label it C.
   d. Draw in segments AC and BC so you can see a triangle ABC inscribed in the circle.
   e. Measure the angles in the triangle, and classify the triangle.
   f. Are there any pairs of complementary angles? If so, name them.
### 5.4b Homework: Review Assignment

Decide if the figures below are possible. Justify your conclusion with mathematical statement. To construct the triangles use:

- A ruler or strips of centimeter graph paper cut to the given lengths
- A protractor
- Construction technology like geogebra

<table>
<thead>
<tr>
<th>Question</th>
<th>Possible or not? Why or why not?</th>
<th>If so, what kind of triangle? Sketch, label.</th>
</tr>
</thead>
<tbody>
<tr>
<td>3. A triangle with sides 8 and 3 cm. The angle opposite the 3 cm side measures 45°.</td>
<td>Possible or not? Why or why not?</td>
<td>If so, what is the measure of the 3rd side? Sketch, label.</td>
</tr>
<tr>
<td>4. A triangle with sides of 8 cm and 3 cm. The angle opposite the 8 cm side measures 45°.</td>
<td>Possible or not? Why or why not?</td>
<td></td>
</tr>
</tbody>
</table>
5. Two students were building a model of a car with an actual length of 12 feet.
   a. Andy’s scale is \( \frac{1}{4} \text{ inch} = 1 \text{ foot} \). What is the length of his model?

   b. Kate’s scale is \( \frac{1}{2} \text{ inch} = 1 \text{ foot} \). What is the length of her model?

6. At Camp Bright the distance from the Bunk House to the Dining Hall is 112 meters and from the Dining Hall to the Craft Building is 63 meters (in the opposite direction). The scale of the map for the camp is \( 0.5 \text{ cm} = 14 \text{ meters} \). On the map,…
   d. …what is the scaled distance between the Bunk House and the Dining Hall?
   e. …what is the scaled distance between the Dining Hall and the Craft Building?

6. In the similar L figures,
   a. What is the ratio of height of left figure : height of right figure?

   b. What is the reducing scale factor?

   c. What is the ratio of area of left figure: area of right figure?

7. Triangle \( ABC \) is similar to \( RST \).
   a. What is the scale factor from \( \triangle ABC \) to \( \triangle RST \)?

   b. What is the scale factor from \( \triangle RST \) to \( \triangle ABC \)?

   c. What is the distance between A and C?

   d. What is the distance between R and S?
8. Redraw the figures at right using the scale factors below.

   a. Use a scale factor of 4 to re-draw the square.

   b. Use a scale factor of $\frac{1}{4}$ to re-draw the addition sign.

   c. Use a scale factor of 1.5 to re-draw the division sign.

9. Bob made a scale model of the Washington Monument using a scale factor of $\frac{1}{1332}$. The height of the model is 5 inches. What is the actual height, in feet, of the Washington Monument?
10. Calculate the circumference and area of the circles below. Express each measurement both exactly in terms of pi, and as an approximation to the nearest tenth of a unit.

a. \( C = \)
\[ C \approx \]
\( A = \)
\[ A \approx \]

b. \( C = \)
\[ C \approx \]
\( A = \)
\[ A \approx \]

c. \( C = \)
\[ C \approx \]
\( A = \)
\[ A \approx \]

11. How many times would a circle with radius 4 fit inside a circle with radius 12, if you could pack the area tightly with no overlapping and no leftover space?

12. Are all circles similar? Justify your answer.

13. Are all squares similar? Justify your answer.
14. Are all rectangles scaled versions of each other? Justify your answer.

15. A circle has an area of $144\pi mm^2$. What is its circumference? Show your work.
16. Find the missing angle measures for the figure below. Justify each answer.

17. Find all the missing angle measures for the figure below. Justify each answer.

18. Draw and label two intersecting lines for which $\angle CDE$ and $\angle ADR$ are vertical angles.

19. Draw and label two intersecting lines for which $\angle HOG$ and $\angle GOX$ are supplementary.

20. Draw and label a pair of adjacent complementary angles $\angle ABC$ and $\angle BCD$. 
5.4c Self-Assessment: Section 5.4

Consider the following skills/concepts. Rate your comfort level with each skill/concept by checking the box that best describes your progress in mastering each skill/concept.

<table>
<thead>
<tr>
<th>Skill/Concept</th>
<th>Beginning Understanding</th>
<th>Developing Skill and Understanding</th>
<th>Deep Understanding, Skill Mastery</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Identify vertical, complementary and supplementary angles.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Find the measures of angles that are vertical, complementary or supplementary to a known angle.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Apply angle relationships to find missing angle measure or knowing angle measures determine angle relationships.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>